

Will China Escape the Middle-Income Trap?

A Politico-Economic Theory of Growth and State Capitalism*

Yikai Wang
University of Oslo

June 2015

Abstract

Is China's rapid growth sustainable with the current institutions? If the growth slows down, will it trigger political changes? To answer these questions, this paper proposes a theory of politico-economic transition of China. In oligarchy, a political elite extracts surplus from the state sector and taxes the private sector. However, to maintain the power, it needs political support from a sufficient number of citizens. A "divide-and-rule" strategy is implemented to guarantee such support: the elite gives state workers high wages to turn them into supporters, and it also distorts the capital allocation in favor of the state sector to maintain enough workers in that sector. The consequences are the following: in the short term, the low wage in the private sector helps private firms and aggregate output grow rapidly. In the mid and long term, the capital market distortions slow down the growth. The theory suggests that the economy develops along an endogenous three-stage transition: *rapid growth* is followed by *state capitalism*, and the economy may follow two paths in the third stage, *middle-income trap* or *sustained growth*, depending on whether democratization occurs. The theory is consistent with salient aspects of China's recent development and gives predictions on China's future development path.

*I am very grateful for helpful comments from Daron Acemoglu, Ufuk Akcigit, George-Marios Angelos, Simeon Alder, Jimmy Chan, Heng Chen, Guido Cozzi, Mikhail Golosov, Xi Li, Nicola Pavoni, Michael Peters, Dominic Rohner, Zheng Song, Kjetil Storesletten, Wing Suen, Jianrong Tian, Nico Voigtlaender, Xiaodong Zhu and Fabrizio Zilibotti. I also thank for thoughtful discussions from participants of various conferences and seminars. The research has received funding from Swiss National Science Foundation (PBZHP1-138697) and ERC grant (324085).

Keywords: rapid growth, state capitalism, middle-income trap, democratization, China, middle-class, financial repression, inequality, state wage premium

JEL Classification: E22 O41 P16

1 Introduction

China has by now been growing at a stellar rate for more than three decades. While this is generally acknowledged to be a great historical achievement, there is major controversy on how far in time and scope the Chinese success story can go. The optimists argue that China can provide a new model for growth as an alternative to the liberal democracy growth model known as the Washington Consensus. For example, in a debate hosted by The Economist (see also Musacchio and Lazzarini (2012)), Aldo Musacchio argues that China's hybrid form of capitalism can become a new growth model for the 21st century. In his view, such a model offers attractive features including less pronounced recessions, focus on long-term investing, and producing world champions. These features make him optimistic about the sustainability of China's future growth, and even the possibility that China could become a role model for other developing and emerging countries. In contrast, pessimists predict that China's growth will soon slow down. For example, Acemoglu and Robinson (2012) argue that China's current institution is not compatible with innovation and sustainable long-run growth, for the following reason. The extractive institution can lead to rapid growth in the early stage, when economic growth is in line with the interest of the ruling elite. However, in the long-run, the elite fears losing its economic rent to new technology or even losing its political power to groups rising from the growth and does not adopt economic arrangements favoring growth. China's growth process driven by catch-up may continue for a while, given the current institution, it will come to a halt as soon as China reaches the living standards of a middle-income country.

The pessimistic perspective of Acemoglu and Robinson raises a number of questions. Will slowing growth, which Acemoglu and Robinson predict, trigger changes in the political system, with unsatisfied citizens outing the oligarchy, and in turn allowing growth to resume under a more democratic system? Or, alternately, will the oligarchy be able to retain sufficient support even in a low-growth economy? Modernization theory suggests that democratization is likely to occur. But, then, one can argue that it may have been right for China to adopt its hybrid form of state capitalism to achieve high economic growth in the catch-up stage, and then switch to liberal democracy when state capitalism runs out of steam. The Chinese model, in other words, could be a model of transition, albeit not a mode of long-run growth for mature economies. This view, however, may

well be overly optimistic: at the time of transition, the political elite could be unwilling to give up state capitalism, and might seek to maintain political power and control of economic resources, as we see in countries like Venezuela. In the language of Acemoglu et al. (2006), state capitalism may be appropriate to promote growth at an early stage of development, but may become impossible to reform when it becomes a barrier to further economic growth.

To answer the above questions - first, whether China's growth can continue, and second, whether changes in political system will occur - this paper proposes a theory of politico-economic transition. A two-sector dynamic general-equilibrium model is built and calibrated to China's economy. The theory is consistent with salient aspects of China's recent developments, including: rapid growth with low wages, large state investments, financial repression on private firms, the support of the middle-class for the government, and so on.

In this theory, a political elite runs the government and is able to extract surplus from state firms and tax the private sector at an exogenous rate. However, it faces a political constraint, that is, support from a sufficient number of citizens. I assume that the government can use the following policy tools to maintain the support: regulating the state sector wage, and controlling capital allocations in the state and the private sector.¹ How does the elite use these tools? First, to buy support from state workers, it sets the state wage sufficiently high - higher than a worker's expected income in democracy. Therefore a dual labor market is created. State workers receive high wages and in turn support the elite. Private worker wages are reduced due to the general equilibrium effect, as follows. Facing high wages, state firms hire less than they could if wages were determined by the market. More workers are pushed to the private sector, and private sector wages are reduced. This "divide-and-rule" strategy gains support from state workers at the cost of private workers. Second, to keep enough supporters in the state sector, the elite needs to balance capital in the state and the private sector. When private firms hold little capital, it is cheap and easy to meet the political constraint, because workers' expected wages and income in democracy are low, and also because private firms hire few workers and the number of workers in the state sector is larger than the number of supporters needed. To extract more tax from the private sector, the elite encourages its growth and helps it to build capital. Once private entrepreneurs get rich and private firms hold more capital, a trade-off emerges: a larger private sector contributes more tax, but it also increases the

¹This means that the government can only adopt clientelism to gain the support, as in Robinson et al. (2013). Other tools, for example, using direct lump-sum transfer to buy the support, are assumed away, following Acemoglu (2003). More discussions on this are in section 3.

cost of maintaining supporters, because it increases the wage and also competes for labor. Then the elite chooses to financially repress the private sector, i.e., to limit its borrowing from banks. This restrains the growth of the private sector capital and relative size.

Because the government policy for the capital market changes as the private sector grows, the economy's growth patterns look different in different stages of development. More specifically, the economy develops along a three-stage transition as follows. The first stage is *rapid growth*, during which the GDP share of the private sector grows rapidly, triggering reallocation and high productivity growth. Private firms benefit from low wages in the private sector, which are induced by the policy. The government supports privatization because it increases tax revenue. However, as privatization continues and the state employment share declines to a critical level, the economy enters the second stage, *state capitalism*. In this stage, the elite over-invests in the state sector to keep the state employment sufficiently large. The government also imposes gradually increasing financial repression to limit the growth of private firms. Growth continues to be fast due to large state investment, but the financial repression on private firms causes a slowdown.

As the private sector capital keeps growing (largely through self finance), two possible outcomes emerge. The first is the *middle-income trap*: over-investment of the state and financial repression of private firms continue, while the efficiency loss also grows, due to decreasing return to capital and the capital market distortion. Eventually, growth stops before the output level converges to the level in democracy. This happens in the case that the cost of retaining the regime is low, e.g., when the number of supporters needed is small. The other possible outcome is *sustained growth*, following democratization. In this case, the cost of maintaining enough supporters in the state sector is high. As the private sector capital grows, the elite finds it too costly to continue investing in the less efficient state sector, and therefore chooses to democratize. Over-investment in state firms and financial repression of private firms both disappear and the economy keeps growing in democracy.

The first two stages in the theory are consistent with the recent development in China. First, low private sector wages helps private firms and the economy grow rapidly. Between 1995 and 2007, the private employment share increased from 40% to 80% (see more details in section 2). This era of fast privatization implies large efficiency gain and *rapid growth*, as in the first stage of the theory. However, the employment share of private sector subsequently stopped growing. Private firms face tighter financial constraints, and around 60% of investment and the majority of bank loans are diverted to less productive state firms.² This capital market misallocation in favor of state firms implies that the

²For instance, Hsieh and Klenow (2009) estimate that the total factor productivity (TFP) of state firms is

economy is entering the *state capitalism* stage. Second, the middle-class, consisting largely of state sector workers and private entrepreneurs, are the beneficiaries and supporters of the regime. This is because state workers receive high wages, and entrepreneurs benefit from the cheap and abundant labor in the private sector. Chen and Lu (2011) and Tsai (2007) document that the Chinese middle-class, including state employees and private entrepreneurs are “achieving their material interests without pursuing any real freedom”. This phenomenon will be discussed in great details in section 2. In addition to explaining the phenomena described above, the theory is also useful for understanding the following phenomena of Chinese economy: high capital labor ratio in the state sector; low and decreasing state sector capital return; high and non-decreasing private sector capital return.

The third stage of the transition in the theory provides an answer to questions about China’s future political and economic developments. The model in this paper, calibrated to China’s economy, predicts that the economy will enter the *middle-income trap*. It is because the government is both economically and politically powerful. In other words, the government’s cost of retaining enough supporters in the state sector is low. First, the government controls abundant financial resource through the banking sector and holds abundant financial assets, including the huge foreign reserve. It is capable of keeping investment in the state sector high and maintaining the current level state employment. Second, the current state employment share, which is not very large, has been sufficient to provide the support that the government needs and keep the political system relatively stable.³ In other words, the government is politically powerful and only needs a relatively small supporter base, i.e., the state sector. Given these conditions, sufficient support for the regime will continue, and policy distortions will persist, which will eventually slow down the growth before China becomes a rich country.

Is China doomed to fall into the middle-income trap? Are there ways to redirect China towards the other development path, i.e. sustained growth? Many China watchers and researchers have proposed reform plans to sustain growth, including political reform, financial reform and state sector reform. However, is the government willing to implement those reforms? Many reforms which are beneficial for growth are harmful for the elite’s interests and thus may not be implemented. Some reforms can be interpreted as changes to the model’s exogenous parameters, and with the aid of the model, I can study consequences of these reforms. A reform can lead to a different development path and can potentially change the third stage to *sustained growth*. The model can also be used to eval-

42% lower than the TFP of domestic private firms.

³The state employment share is about 20% in the urban area, according to the statistical yearbook 2014.

uate how a reform affects the elite's interest, and to think on which reforms are likely to be implemented or face strong resistance from the elite. This analysis is useful for predicting China's future policies and the direction of reforms.

This paper is related to three strands of literature. The first is on China's economic growth through resource re-allocation. Song et al. (2011) construct a two-sector growth model to study how the capital and labor reallocation from the state to the private sector leads to rapid growth. Brandt and Zhu (2000, 2010) document the contribution of private firms to growth and the government's strategy of maintaining state sector employment. These studies capture some key features of China's economic growth, including the capital and labor market frictions. However, an important unanswered question exists in the previous research: why do large labor and capital market frictions exist and how will they evolve in the future? To answer this question, political economy needs to be modeled, as it is the root of frictions, including the financial constraint on private firms in Song et al. (2011) and the state employment constraint in Brandt and Zhu (2010). This paper provides the micro-foundation for the endogenous evolution of state employment, labor market wedge, and financial constraint. It not only helps to better explain the frictions, and more importantly, allows us to predict their future trends. In contrast to the conventional wisdom which suggests that these frictions will gradually decline as China's labor market and financial market become more mature, this paper predicts that they will be persistent and will even increase.

Second, our theory contributes to the study of middle-income trap, i.e., the significant slowdown of economic growth when a country's GDP per capita reaches the middle level. Gill and Kharas (2007) is the first paper that formally uses this term to describe the growth slowdown of emerging economies. In empirical studies, Eichengreen et al. (2013) document this pattern with data, and Robertson and Ye (2013) propose a statistical definition of the middle income trap, namely, the convergence of a country's relative income to the U.S. to a level in the range of 8% to 36%. Robertson and Ye (2013) test the definition with a cross-country panel data and find that out of 46 middle income countries, 19 fall into middle income trap. While the discussion of the middle-income trap is popular in public and academia, there is a lack of theoretical frameworks to aid the discussion. This paper tries to formally model how an economy can grow rapidly within the extractive institution, why the growth may stop when the economy is at the middle-income level, and how reforms on political institutions can further sustain growth. Moreover, the theory allows us to study under which conditions a country can jump out of the middle-income trap and which reforms are necessary to achieve this.

The third strand of literature is on the relation between political development and eco-

conomic development. Acemoglu and Robinson (2012) explain how political institutions affect economic performance in the long run. They argue that the *extractive* political institution is detrimental to long-run growth. On how economic development affects political development, the modernization theory, originated from Lipset (1959a) suggests that the economic development will ultimately lead to political modernization, i.e., liberal democracy. This paper's contribution to this strand of literature is two-folds. First, it combines both sides of the relations and studies the interactions of them - not only how political institution affect economic development but also how the later determines the former. This is done by incorporating political economy into a neo-classic growth model. This way of modeling can be useful for researchers who are interested in both political economy and growth. Second, the theory distinguishes the short-term effect of political institution on economic development from the long-term effect and suggests that institutions that help rapid growth in the catch-up stage can be detrimental to growth in the long run.

The rest of the paper is organized as follows. Section 2 shows important empirical facts on China's political-economic development that motivate the theory. Section 3 discusses a two-sector dynamic growth model with the three-stage political-economic transition. The first two stages explain important phenomena and puzzles in China's recent development, while the third stage predicts future politico-economic trend. Section 4 concludes.

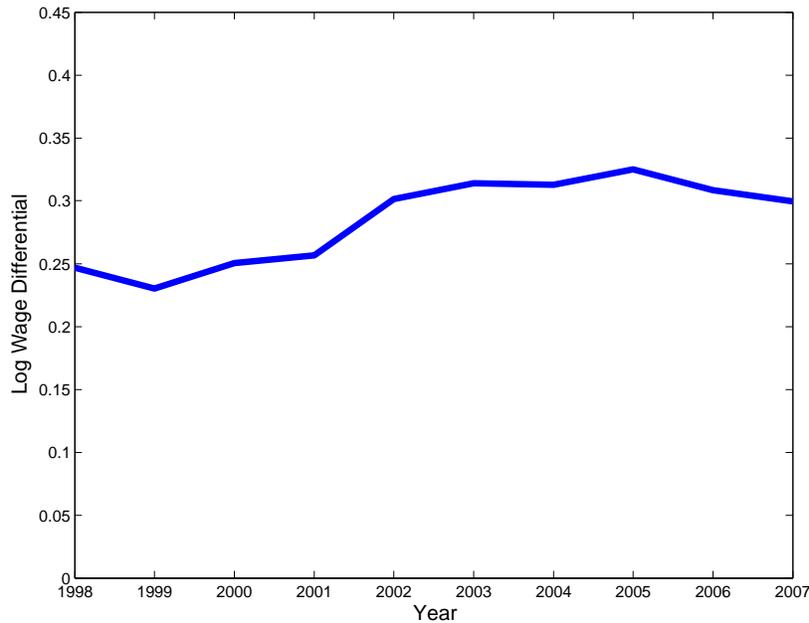
2 Empirical Facts on China's Recent Development

In this section, the following key facts in China's recent development that motivate the theory are documented: (1) large wage gap between the state and the private sector; (2) low support for democracy from the middle-class; (3) the slow-down and stop of privatization; and (4) financial market wedges between the state and the private firms.

2.1 Large State-Private Wage Gap

China's state workers enjoy a wage premium of around 20% to 30%, as documented by Ge and Yang (2014). Their finding is based on a Mincer regression controlling for observable characteristics - age, education, industry, region and so on, and their result is reproduced in figure 1. In contrast, the wage premium of state workers in Canada, Germany and the U.S. are estimated to be lower than 5% or insignificant after the 1990s. See Melly (2002), Mueller (1998) and Poterba and Rueben (1994).

Figure 1: State Sector Wage Premium



Source: Ge and Yang (2014).

2.2 Middle-class Support for the Regime

Given that the state workers are earning high wages, it is not surprising to see that they are more supportive of the current political system compared to non-state workers, as documented by Chen and Lu (2011). The authors use survey data of 2810 individuals, collected in three Chinese cities in late 2006 and 2007 to estimate how the individual's political opinions depend on his or her characteristics, especially the social group identity. They find that state sector workers and the middle class are less supportive for democratic values. For example, only 24.9% of the middle class support multi-party competition, while 38.7% of the lower-class do. Only 22.9% of the middle class agree that demonstrations should be allowed, while this number is 35.6% for the lower class.⁴ Similar patterns apply for other questions related to democratic values and institutions. To formally show the difference between the middle class and the lower class, the authors combine answers to multiple questions into one index of support for democratic values and institutions us-

⁴The authors define class according to the employment status. Individuals with jobs which usually pay low wages are classified as the lower class, including blue-collar workers, unemployed and self-employed with very little capital. The middle class mainly consist of white-collar workers. They distinguish private entrepreneurs from the middle class, while private entrepreneurs are usually considered an important part of the middle class. The authors also report that private entrepreneurs hold similar political opinions as the middle class.

ing factor analysis.⁵ Then they run a regression of this index on individual characteristics, including a dummy for middle-class membership and a dummy for state employment. The coefficients of dummies for middle-class and state employment are both negative (-1.23 and -0.54) and significant at at 1% level. In contrast, party membership does not affect the political opinion too much, after controlling for other variables. The coefficient of the party membership in the regression is -0.37 and not significant at 5% level. These suggest that economic interest plays a more important role than ideology. In other words, the middle class, including many state sector workers, are more supportive for the current political system. In another paper, Tsai (2007) documents that the Chinese entrepreneurs are “achieving their material interests without pursuing any real freedom,” in contrast to “the business classes in historical England, France and the United States” who “have risen up against the government to defend material interests. ”

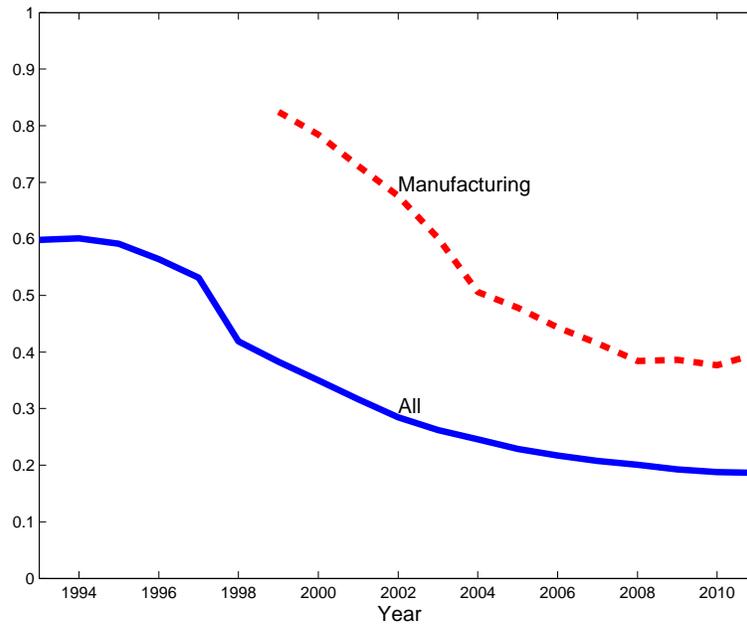
In short, the Chinese middle class, consisting largely of state workers and private entrepreneurs do not support democracy, contrary to the conventional wisdom that the middle-class are the driving force for democratization.

2.3 The Slow-Down and Stop of Privatization

If state workers support the government while many state firms are not productive, will the government allow the state employment to decline? The answer is mixed: initially the government allows the state employment to decline in order to improve the efficiency of the economy, but it does not allow the state employment to become too low, because state employees are an important supporter base. The decline of state employment and the privatization of state firms was very rapid for a couple of years, after the fifteenth national congress in 1997, which initiated the state firm reform. Many inefficient state firms bankrupted or got privatized, while many private firms entered the market and grew rapidly. As the blue line in figure 2 shows, the employment share of state sector in urban areas declined rapidly from 53% in 1997 to 23% in 2005. After that, the privatization slows down and the state employment share stagnated at around 20%. If we focus only on the manufacturing, mining, and construction, represented by the red line, the trend is similar while the state employment share stops declining at a higher level around 40%, and even slightly increases after 2011. This is the so-called “the state advances as the private sector retreats” phenomenon and it suggests that the privatization and the decline

⁵The survey data contain four dimensions of questions on support of democratic values, including right consciousness, valuation of political liberty, support for participatory norm and support for competitive election. The index for support for democratic values and institutions is the constructed as the single dominant factor using factor analysis.

Figure 2: State Employment Share in the Urban Sector



Source: Statistical Yearbook 2012 and Storesletten and Zilibotti (2014).

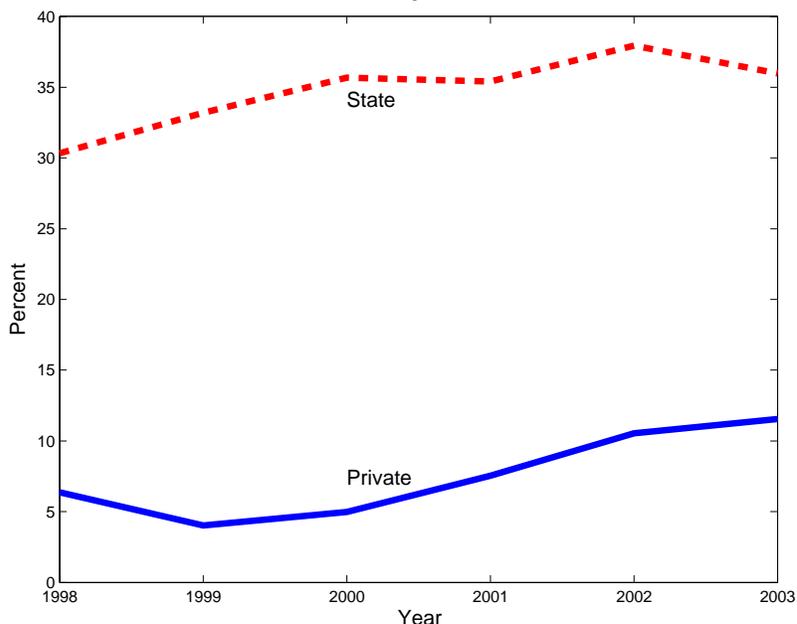
of state employment has come to a halt. Moreover, there is more direct evidence that the government intentionally keep the state sector alive. For example, the closing announcement of the Third Plenary Session of 18th Chinese Communist Party Central Committee in 2013 stated that “China will stick to the dominant role of public ownership, playing the leading role of the state-owned economy, while encouraging, supporting, and guiding the non-public sector.”

2.4 Capital Market Wedge between the State and the Private Sector

How do state firms survive and hire a significant fraction of workers, if they are much less efficient than private firms, as documented Hsieh and Klenow (2009)? State firms survive because they are in a more advanced position in the financial market compared to private firms. It is easier and cheaper for state firms to get loans from state banks compared to private firms. Song et al. (2011) document, as shown in figure 3, that while state firms finance more than 30% of their investment through bank loans and government budget, this number is less than 10% for private firms. Their result is reproduced in figure 3. Brandt et al. (2012) estimate that the capital wedge, i.e. the ratio of costs per unit of capital between state and private firms, has increased in all the provinces, on average from 4.2 in 1996 to 6.8 in 2007.

Is the capital wedge between state and private firms due to China’s immature financial

Figure 3: Share of Investment Financed by Bank Loans and Government Budget



Source: Song et al. (2011).

market so it will decline as the financial market develops over time? Or alternatively, is the capital wedge maintained by the government to keep the state sector large enough, and will the government strategically keep it or even increase it in the future? In the next section, we can study these questions with the help of a general equilibrium growth model with political constraints. The model is also used to explain other facts discussed in this section, including the state wage premium, middle class's support for the regime, and the decline of state employment share. Moreover, the model is used to predict the future trend of these phenomena, as well as economic growth, political transition.

3 The Model

This section presents a theory of politico-economic transition to address the questions and understand the key phenomena discussed above. I build a two-sector dynamic general equilibrium growth model which incorporates the political constraint and the political choices of agents to study the interactions between political and economic developments. I first discuss the general properties of the model and then study its implications for China with the aid of a calibrated economy.

3.1 Preferences and Technology

The model economy is populated by three classes of agents: an elite (e), private entrepreneurs (p), and workers (w). Each class consists of infinitely many members. The population of workers is normalized to measure 1, and the population of the elite members and private entrepreneurs are both assumed to be small and of measure 0.

There are two sectors and two types of firms. State (S) firms produce in the state (S) sector, while private (P) firms in the private (P) sector. There are infinitely many of them. They produce the same final goods using capital and labor and maximize profits. They are different in two aspects, ownership and productivity. Each S firm is owned by an elite member, while each P firm by a private entrepreneur. S firms are less productive than P firms. The technology of S and P firms is described by the following production functions:

$$\begin{aligned} Y_S &= (z_S K_S)^\alpha L_S^{1-\alpha}, \\ Y_P &= K_P^\alpha L_P^{1-\alpha}, \end{aligned}$$

where $z_S < 1$, K_S, K_P are S and P sector capital while L_S, L_P denote for S and P sector labor, respectively.

The elite provides capital to S firms and entrepreneurs provide capital to P firms. They earn income from the capital returns. The elite and entrepreneurs live for infinite periods, and maximize lifetime utilities. Their instantaneous utility is assumed to be logarithmic and the discount factor is denoted by β . Workers provide 1 unit of labor inelastically. For simplicity, I assume that workers live hand-to-mouth and they are myopic, i.e., they consume all the income every period, and in each period they care only about current period income.

The elite has access to the deep pockets of banks. It can borrow from banks and set S sector capital without constraint. An entrepreneur finances firm capital using her asset and bank loans. However, she faces a financial constraint: the bank loan cannot exceed $\eta - 1$ fraction of her asset. In other words, the P firm leverage (ratio of capital over net asset) is bounded above by η . η is set by the government within an interval: $\eta \in [\underline{\eta}, \bar{\eta}]$. Furthermore, I assume banks can borrow and lend in the international bond market at the interest rate r and compete with each other, so the interest rate for bank loans is simply r , and the interest rates for savings of the elite and entrepreneurs in the bank are also r .

The setting on the financial market is similar to Song et al. (2011), which also assume that the state firms have unlimited access to bank loans while private firms face financial constraints. The key difference is that here I allow the financial constraint - P firm leverage η - to be endogenously determined by the government. The government can

create barriers for loans to private entrepreneurs, and can directly give administrative instructions to banks (see Brandt and Zhu (2000)). $\underline{\eta}$ is the lower bound of the leverage. For example, $\underline{\eta}$ equals 1 means that the strictest policy that the government can set is to order banks not to lend to private entrepreneurs at all. Then the entrepreneurs can still finance their investment using their own assets. $\bar{\eta}$ is the highest leverage if the government doesn't restrict private sector financing at all. The upper bound for the leverage can be thought as the consequence of a moral hazard problem, as in Song et al. (2011), i.e., an entrepreneur with too high level loan compared to her asset chooses not to repay the loan and run away.

3.2 Political Systems and the Government

There are two types of political regimes: democracy and oligarchy.

In democracy, the government is elected by the majority vote. Hence a representative worker runs the government forever, given the dominating size of workers. I assume that the government collects taxes from the ruled groups, i.e., the elite and entrepreneurs, and transfer tax income equally to the ruling group - workers. The tax rate is exogenously given as $\tau^D > 0$.⁶ The capital and labor markets are competitive. Each entrepreneur decides capital supply to P firms subject to the financial constraint $K_P \leq \bar{\eta}a_p$, while each elite member chooses capital supply to S firms. Each worker supply labor to a S or P firm. In other words, democracy implies that in each period, the economy is in a competitive equilibrium given taxes on the elite and entrepreneurs. The distortion in democracy is simply because of the capital tax, and there are no other distortions in the capital and the labor market. This setting is in the spirit of Acemoglu (2008).

In oligarchy, the elite controls the government, but faces a political constraint, that is, it needs political support from a sufficiently large fraction of workers. Each worker, after being employed by an S or P firm and observing the government policies, decides to support the oligarchy or not based on the expectation on her income in this period. Oligarchy is sustained if more than \underline{L} workers choose to support it. If less than \underline{L} workers' support oligarchy, democratization occurs. Moreover, the government collects taxes from entrepreneurs and private firm workers and then transfer to the elite. The tax rate is ex-

⁶Alternatively, one can endogenize the tax decisions while still getting the equivalent results, as follows. Suppose that the government decides which groups to tax and tax rates, and tax payers can hide their income at the cost of τ^D fraction of the income. Then, the government optimally taxes the elite and the entrepreneurs at the highest rate: τ^D and transfers the tax income to workers, because it cares only about workers' current period income. This is a simple way to endogenize tax rates, as used in Acemoglu (2008) and referred as "state capacity" in Besley and Persson (2009).

ogenously given as $\tau > 0$.⁷⁸ The capital and labor markets are no longer in a competitive equilibrium - now the government sets S sector minimal wage w_S , S sector capital K_S , and P sector leverage η . Given government policies, S firms compete for capital in the S sector, and hire labor obeying the S sector minimal wage to maximize profits. P firms compete for capital in the P sector supplied by entrepreneurs.

Next, let me explain the logic for the settings in oligarchy. First, the micro-foundation for \underline{L} is based on Acemoglu et al. (2012), as follows: if the elite and their supporters form a coalition which has large enough political power, oligarchy is sustained. More specifically, a coalition of a set of agents holds a corresponding level of political power. If the political power of a certain coalition is large enough, it can choose the political system. In this paper, under oligarchy, the elite as the ruling group is granted political power ω_e . Each worker has political power ω_w , and each entrepreneur has ω_p . The aggregate political power of entrepreneurs is 0 given its size of 0. Workers can change the political regime from oligarchy to democracy if and only if they form a coalition of size L_r whose power is larger than α , namely $\frac{\omega_w L_r}{\omega_w + \omega_e} > \alpha \Leftrightarrow L_r > \alpha \frac{\omega_w + \omega_e}{\omega_w}$, where α is exogenous. In other words, to sustain the oligarchy, there must be at least $1 - \alpha \frac{\omega_w + \omega_e}{\omega_w}$ workers supporting it. This size is denoted as \underline{L} . Notice that \underline{L} captures the relative political power of the elite compared to workers. If the elite is very powerful, it needs only a small fraction of workers as supporters to form a winning coalition. If workers are well-organized and politically motivated, \underline{L} becomes large. Second, the government sets state capital K_S , and private sector leverage η . In China, the government controls state banks, and thus is able to influence the allocation of loans to private and state firms. It can directly invest in state firms or set barriers for private firms getting bank loans. Or, it can subsidize the capital of state firms to influence their investment, e.g., providing cheap loans to state firms in certain industries. So alternatively but equivalently, one can also think this model as that the government sets the interest rates in the state sector. The current setting, that the government directly set K_S and η , is more straightforward.

Notice that in this setting, two important assumptions are made. First, state firms maximize profits in oligarchy and choose employment. The government can only use prices to influence the state employment by regulating the wages in the state sector, but

⁷⁸Similar to the case in democracy, tax decisions can be endogenized. The elite optimally chooses not to tax S workers to make it easier to buy their support. The elite may or may not set the tax rate on entrepreneurs to the highest possible level, depending on how much asset entrepreneurs hold. However, in our calibrated model, the numerical solution with endogenous tax rate decision is that the elite always optimally chooses to tax entrepreneurs and P workers at the highest rate τ , so it is equivalent to the simple setting of exogenous tax rate.

⁸Notice that we use τ instead of τ^O , to simplify the notation. We drop the superscript O for variables in oligarchy when there is no confusion.

can not directly set labor in each state firm. This is consistent with the current situation of state firms in China. After a series of state sector reforms, especially after the 15th Party Congress in 1997, to improve the efficiency of state firms, power has been gradually delegated to firms from central and local governments. Now state firms are incentivized to maximize the profits and are allowed to make operational decisions, including hiring workers. The government can influence decisions of state firms through the price system. The second assumption is that the government can not make direct transfer to the ruled groups to buy political support. Though direct transfer seems to be a cheap and attractive way to buy support, in the literature of political economy, it is considered as difficult to implement, for two reasons. One is the credibility and commitment problem. Acemoglu (2003) and Acemoglu and Robinson (2005) argue that even if the state promises to make a transfer to the ruled group, the latter, without political power, gets no guarantee that it will eventually receive the transfer. The promise of transfer is not credible, and transfer cannot be used to solve all the political conflicts. The other reason direct transfer is difficult to implement is the high cost due to local capture. This is supported by empirical evidence. Reinikka and Svensson (2004) document that 87% of the transfer from the central government to local schools in Uganda was not received during 1991-1995 due to local capture. This means that the cost of 1 dollar of transfer is as high as 7.7 dollars. For these two reasons, the government usually builds inefficient “white elephant” projects (see Robinson and Torvik (2005)) to guarantee the economic benefits for certain groups. In our model, state firms can be considered as a special type of “white elephants”.

3.3 Equilibrium and Aggregate Dynamics

Given the settings described above, the rest part of this section presents the solution of the model. The dynamic equilibrium consists of infinite periods, and each period can be separated into three stages: (1) determination of capital in S and P sectors, (2) political outcome and the equilibrium of the labor market in this period given capital allocation, and (3) decisions on consumption and saving. In this subsection, I first focus on stage (2) of each period and study the political and economic outcomes given capital allocation.

3.3.1 Equilibrium Given Capital Allocation

In democracy, the labor market is competitive. Wages in S and P firms are the same and are equal to the marginal productivity of labor:

$$w^D = (1 - \alpha) (z_S K_S)^\alpha (L_S^D)^{-\alpha} = (1 - \alpha) (K_P)^\alpha (L_P^D)^{-\alpha}.$$

A worker's income equals the wage plus the tax collected from entrepreneurs and the elite:

$$\begin{aligned} y_w^D &= w^D + \tau^D (\pi_S^D + \pi_P^D) \\ &= \left(1 + \tau^D \frac{\alpha}{1-\alpha}\right) w^D, \end{aligned}$$

where π_S^D and π_P^D are the capital incomes of the elite and entrepreneurs from S and P sectors, respectively, and τ^D is the tax rate in democracy. Notice that to simplify the expressions, I assume that the taxes are applied to the raw capital incomes but not capital incomes net of depreciation and interest payment. The transfer to workers is $\tau^D \frac{\alpha}{1-\alpha} w^D$ simply because the tax base, capital income, is $\frac{\alpha}{1-\alpha}$ times labor income.

In oligarchy, the following events happen sequentially: (1) the government sets S sector minimal wage; (2) S and P firms hire workers; (3) S and P workers decide whether to support the current political system; (4) the share of supporters determines the political outcome, i.e., if oligarchy does not get enough support, the economy switches to the equilibrium in democracy; (5) firms produce, labor and capital incomes are distributed; (6) the government collects tax and makes transfer.

First, the government chooses S sector minimal wage w_S to influence the labor market outcome and the economic benefits of S and P workers. I can safely only consider the cases where $w_S \geq w^D$ so the minimal wage constraint is tight.⁹ Given the minimal wage, the representative S firm chooses labor demand L_S such that wage equals marginal productivity:

$$w_S = (1 - \alpha) (z_S K_S)^\alpha L_S^{-\alpha}. \quad (1)$$

Remember that the oligarchic government cannot use direct transfer to buy political support, so the final income of S workers is simply $y_{w_S} = w_S$. If the government wants to increase S worker income, it has to set a high S sector minimal wage, which distorts the labor market. We can see this in figure 4. Red and blue lines are the marginal productivities of labor in S and P sectors, respectively. The intersection of the two lines pins down the equilibrium in democracy: the S sector labor, wage and worker income in democracy are denoted as L_S^D, w^D and y_w^D . In oligarchy, w_S pins down S sector labor and its marginal productivity. The rest of labor is in the P sector and pins down the P sector wage w_P . Setting w_S greater or equal to y_w^D implies that the marginal productivity of S sector labor is greater or equal to y_w^D and the S firms hire less or equal to \bar{L} .

⁹Setting minimal wage $w_S < w^D$ is equivalent to setting $w_S = w^D$.

workers support oligarchy. Then high enough state wage is equivalent to low enough state employment share $L_S \leq \bar{L}$. The second is the *minimal support constraint*, i.e., $L_S \geq \underline{L}$. The government faces a critical labor market trade-off between these two political constraints: a high w_S buys S workers' political support and guarantees high state wage constraint while it implies a low level of S sector employment L_S , which may violate the minimal support constraint.

If the government can choose w_S such that the two constraints are both satisfied, oligarchy is sustained. However, it is not always true that both constraints can be satisfied at the same time. This depends on the capital allocation between S and P sectors. \underline{L} is an exogenous parameter, determined by political power of workers and the elite. \bar{L} is endogenously determined by y_w^D , which depends on the capital allocation K_S and K_P , as follows:

$$w_S = (1 - \alpha) K_S^\alpha L_S^{-\alpha} \geq y_w^D = \left(1 + \tau^D \frac{\alpha}{1 - \alpha}\right) (1 - \alpha) K_S^\alpha (L_S^D)^{-\alpha} \Rightarrow$$

$$L_S \leq \nu L_S^D = \nu \frac{zK_S}{zK_S + K_P} \doteq \bar{L},$$

where $\nu = \left(1 + \tau^D \frac{\alpha}{1 - \alpha}\right)^{-\frac{1}{\alpha}}$. If $\frac{zK_S}{K_P}$ is large enough, \bar{L} can be larger than \underline{L} . In other words, sustaining oligarchy requires that S sector is equipped with enough capital, relative to the P sector capital. The equilibrium is summarized in the following.

Proposition 1 (Equilibrium given capital allocation). *If there is sufficiently large capital in S sector relative to the capital in P sector, oligarchy can be sustained. Wage and capital labor ratio in S sector are higher than in P sector while capital return in S sector is lower. Capital return and entrepreneur income in P sector are higher than in democracy. If S sector capital is small, oligarchy cannot be sustained.*

In S sector, the capital labor ratio is high and capital return is low because of the high wage and low level of labor, as shown in 4. In P sector, because of the low wage and the abundant labor, capital return is high. In this case, the one period elite income is the following:

$$y_e = \pi_S - (r + \delta) K_S + \tau w_P L_P + \tau \pi_P, \quad (2)$$

where $\pi_S = \alpha (zK_S)^\alpha L_S^{1-\alpha}$ and $\pi_P = \alpha (K_P)^\alpha L_P^{1-\alpha}$ are capital incomes of S and P firms, respectively. An entrepreneur's income is

$$y_p = (1 - \bar{\tau}) \pi_P - (r + \delta) K_P.$$

3.3.2 The Dynamic Equilibrium

Because of the importance of economic power, the government is motivated to control capital accumulation and allocation between the state and the private sector. In this following, we study the dynamic equilibrium, including the allocation of state and private sector capital, consumption, and saving, based on the equilibrium given capital allocation discussed above.

In democracy, workers control the government to maximize their income. The government does not want to change the political system, and since there is no political constraint in democracy, democracy continues forever in this model. Given the assumption in the model, the economy in democracy is a competitive equilibrium given capital. The dynamics in democracy is basically a two sector growth model in which resource are gradually reallocated from the inefficient sector to the efficient one, as in Lewis (1954) and Song et al. (2011). The dynamic equilibrium in democracy is summarized in the following.

Proposition 2 (Dynamic equilibrium in democracy). *In democracy, an elite member gets return on her asset at interest rate r . Her income only comes from her asset and income net of asset return is 0. An entrepreneur saves β fraction of her total resource - asset plus asset return - at the end of each period, . If β is large enough, entrepreneur assets increase over time. Gradually, the relative size of S sector over P sector, measured by $\frac{K_S}{K_P}$, decreases to 0 .*

The intuition for the above result is the following: efficient labor allocation implies the same wage in S and P sector. S firms compete in the S sector capital market, so the capital return equals the cost of financing, i.e., the interest rate r at which elite members can borrow from banks or international financial market freely. The capital return pins down S firm capital labor ratio and wage. P firms hire workers at the same wage rate as S firms, but they are more productive, so P firm capital return is higher. Entrepreneurs, however, face financial constraint, so if entrepreneur asset and P firm capital are small, P firms can't hire all the workers and S firms still exist . In this case, entrepreneurs get a return higher than r from their asset. If β is large enough, entrepreneurs savings increase over time, entrepreneur assets and P firm capital increase over time, and finally P firms hire all workers and S firms all exit. Market force is decisive in such a competitive equilibrium and the inefficient S firms gradually get replaced by P firms..

In oligarchy, the representative elite controls the government and decides on three policies: minimal wage in S sector w_S , S sector capital K_S , and P firm leverage η . Given these policies, the choices of workers and entrepreneurs are simple. S and P workers behave as in subsection 3.3.1, i.e., support oligarchy if and only if their income is higher

than in democracy. Then they consume all the income.

Each entrepreneur, being a small agent, takes the political outcome and P sector capital return as given, and maximizes lifetime utility. She applies for bank loans, lends to P firms, consumes, and saves for the future. Because an entrepreneur's income only comes from asset return, and the rate of return does not depend on her asset holding but the equilibrium price, her optimal choice is quite simple, just as as stated in the following lemma and proved in the appendix.

Lemma 1. *In oligarchy, if the P firm capital return is higher than the interest rate of bank loan, an entrepreneur's optimal choice can be separated into two steps. First, she borrows as much as possible and invests all into P firms, to maximize her current period income; then, she saves a constant fraction of this period's total wealth and consumes the rest, to maximize her lifetime utility.*

In each period, the representative elite's dynamic problem contains two parts. First, she chooses to sustain oligarchy or to democratize:

$$W(a_e, a_p) = \max \left\{ W^O(a_e, a_p), W^D(a_e, a_p) \right\}, \quad (3)$$

where W stands for lifetime utility given elite asset a_e and entrepreneur asset a_p , while W^O and W^D are the lifetime utility given that the elite chooses to stay in oligarchy or to democratize, respectively. If the later is chosen, the economy ends up in the dynamic equilibrium of democracy discussed above. If the former is chosen, in the next step, she picks government policies η, K_S, w_S, τ_p to sustain oligarchy.¹⁰ She also decides consumption and saving to maximize her lifetime utility.

$$\begin{aligned} W^O(a_e, a_p) &= \max_{w_S, K_S, \eta, c_e, a'_e} \log c_e + \beta W(a'_e, a'_p) \\ \text{s.t. } w_S &\geq y_w^D(K_S, \eta, a_p), \\ L_S &\geq \underline{L}, \\ a'_e &= Ra_e + y_e(w_S, K_S, \eta, a_p) - c_e, \\ a'_p &= \beta (Ra_p + y_p(w_S, K_S, \eta, a_p)), \end{aligned} \quad (4)$$

where $R = 1 + r$. From the expression of y_e in equation 2, we can see that within each period a_e only contributes to the elite's income through interest revenue and does not constraint or affect other equilibrium choices at all. It also doesn't directly affect future state

¹⁰Here we can safely assume away the possibility that in the first step the elite chooses oligarchy, but then in the second step it sets policies that cannot sustain oligarchy, to simplify the notation. In the calibrated economy and the numerical solution, I can allow the elite to choose this, but it never does so.

variables a'_p and a'_e . The contribution of a_e is simply Ra_e in the elite's budget constraint. Its only role is consumption smoothing. Therefore the representative elite's problem, similar to an entrepreneur's problem, can be separated into two sub-problems, as the following lemma states.

Proposition 3. *In oligarchy, the representative elite's optimal choices can be separated into two sub-problems. First, maximization of the lifetime income with discounting rate $\frac{1}{1+r}$ by choosing government policies. Second, maximization of the lifetime utility using a_e to smooth consumption.*

The second sub-problem is straight-forward and does not affect the first one or the politico-economic outcomes. The first sub-problem therefore has only one state variable, as follows:

$$\begin{aligned} V^O(a_p) &= \max_{w_S, K_S, \eta} y_e(w_S, K_S, \eta, a_p) + \frac{1}{R} V(a'_p) \\ \text{s.t. } w_S &\geq y_w^D(K_S, \eta, a_p), \\ L_S &\geq \underline{L}, \\ a'_p &= \beta(Ra_p + y_p(w_S, K_S, \eta, a_p)), \end{aligned} \quad (5)$$

where V^O stands for the part of discounted lifetime income of the elite which is not related to its asset a_e . Continuation value $V(a'_p)$ depends on the political outcome of next period, as follows

$$V(a_p) = \max \left\{ V^O(a_p), V^D(a_p) \right\},$$

where $V^D(a_p) = 0$ because the elite income in democracy is simply the asset return Ra_e , as stated in Proposition 2.

The dynamic equilibrium does not allow for analytic solution, so we will solve it numerically in the next subsection.

3.4 Quantitative Analysis

In this subsection, I calibrate the model to the Chinese economy and solve the model numerically. The properties of the model will be studied, and simulation of the dynamics will be done, in order to think about China's development in the past and in the future. The targets of the calibration are the key facts in China's recent development, including the wage gap, speed of privatization, and the state employment share.

3.4.1 Calibration

The economic parameters are set as follows. First, the production function is Cobb-Douglas with the capital share $\alpha = 0.5$ (Bai et al. (2006)) and depreciation rate $\delta = 0.1$ (Song et al. (2011)). The state capital efficiency is set to be half of the private capital $z_S = 0.5$. This implies that the TFP of state firms is 71% of the TFP of private firms. This is higher than 59% estimated by Hsieh and Klenow (2009) with data before 2005, but is reasonable considering that the trend of declining TFP gap discussed in Hsieh and Song (2013b). Second, the interest rate of bank saving is $r = 5\%$. Third, the discount factor of entrepreneurs, which is also the saving rate of their lifetime income, is set to $\beta = 0.9$ to match the rapid private sector employment share growth from around 40% to around 80% in 5 years, as we can see from figure 2. Finally, the tax rate is set to $\tau^D = \tau = 20\%$ to match the state-private wage gap of 30%, as in figure 1. The political parameter in this model is \underline{L} , the minimal support needed to sustain oligarchy. We set $\underline{L} = 20\%$, as the state employment share converges to around 20% as in figure 2.

3.4.2 Numerical Solution

In the following, I explain the properties of the numerical solution of the elite's dynamic programming problem, in three steps: (1) given K_S and K_P , the choices of other variables; (2) given K_P , the choice of K_S ; and (3) the choice of η that affects K_P .

First, given K_S and K_P , we know from subsection 3.3.1 that if K_S is large enough, there is some w_S that sustains oligarchy, or equivalently, some L_S that falls into the region $[\underline{L}, \bar{L}(K_S, K_P)]$. Generally, the optimal choice of w_S is y_w^D , or equivalently, $L_S = \bar{L}$. This choice implies the least labor distortion but still satisfies high state wage constraint.¹¹ In other words, the elite prefers not to distort the labor market more than the necessary. Furthermore, τ_P is generally set at the highest level $\bar{\tau}$.

Second, how does the government choose K_S , given K_P ? In figure 5, we use a numerical example to depict how state sector labor, political outcome, elite income, and marginal benefit of state capital for the elite depend on the choice of K_S (the x-axis). Given a K_P , if K_S reaches a certain critical level, there can be enough state workers (left-upper panel) - in this figure, $\underline{L} = 0.2$ - and oligarchy can be sustained (right-upper panel). Then there is a jump in elite income above the critical level of K_S (left-lower panel) because the elite controls the government and the tax system in oligarchy. For this reason, though the cap-

¹¹This is true as long as the tax rate $\bar{\tau}$ is not too high. One sufficient condition is $\bar{\tau} \leq \alpha$, which is a reasonable constraint, considering that α is estimated to be around 0.5 in China. If $\bar{\tau}$ is too large, the elite can extract more from the private sector than from the state sector, the solution may change, but this is not very reasonable.

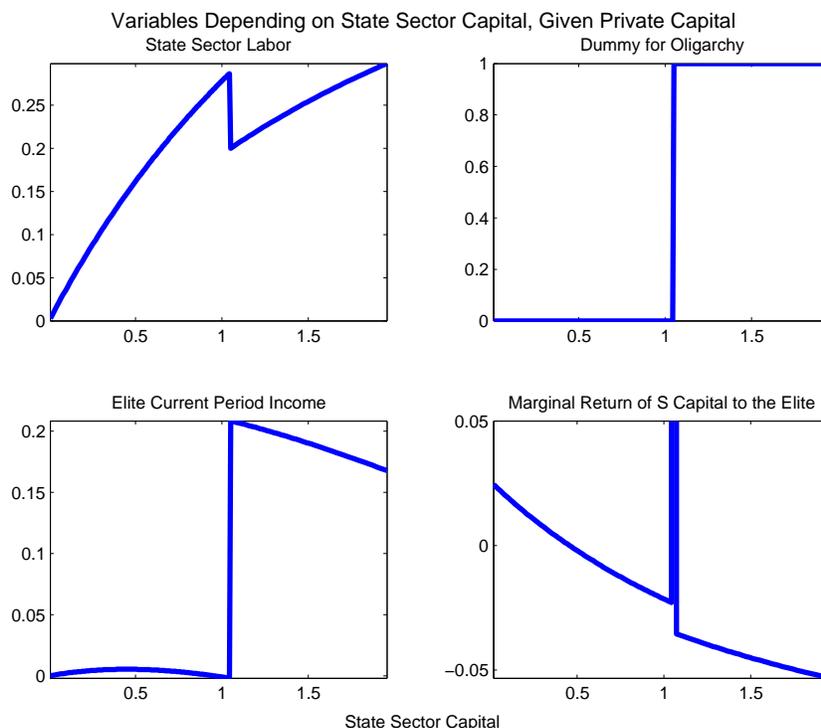


Figure 5: The outcome depending on choice of K_S

ital return goes even lower than 0 as more capital is invested in the state sector, the elite still prefers to invest until the critical level of S capital K_S (right-lower panel) to sustain oligarchy.

In the example above, given its particular level of P capital K_P , choosing S capital K_S that just sustains oligarchy gives highest current period income to the elite. But for other levels of K_P , the situation may be different. As we can see in figure 6, when K_P (the x-axis) is very small, K_S is negatively related to K_P (left-upper panel) and L_S is larger than \underline{L} (right-upper panel).¹² In this region, a larger K_P , corresponds to a larger P sector labor and a smaller S sector labor, hence it is optimal for the elite to reduce investment in S sector - K_S - accordingly. However, when K_P is large enough, and S sector labor reaches the minimal level \underline{L} , a larger K_P implies that the government has to invest more in S sector to maintain oligarchy. We can see that a larger P sector not only increases benefit for the elite - tax income, but also creates higher cost - larger interest payment for K_S (left-lower panel). Due to the decreasing return to capital, there is a level of K_P that maximizes the elite income (right-lower panel).

How does the elite set K_P to be closer to the optimal level for the elite? In the third step here, we discuss the choice of η that affects entrepreneurs' borrowing ability and

¹²Figure 6 comes from the same numerical example as figure 5.

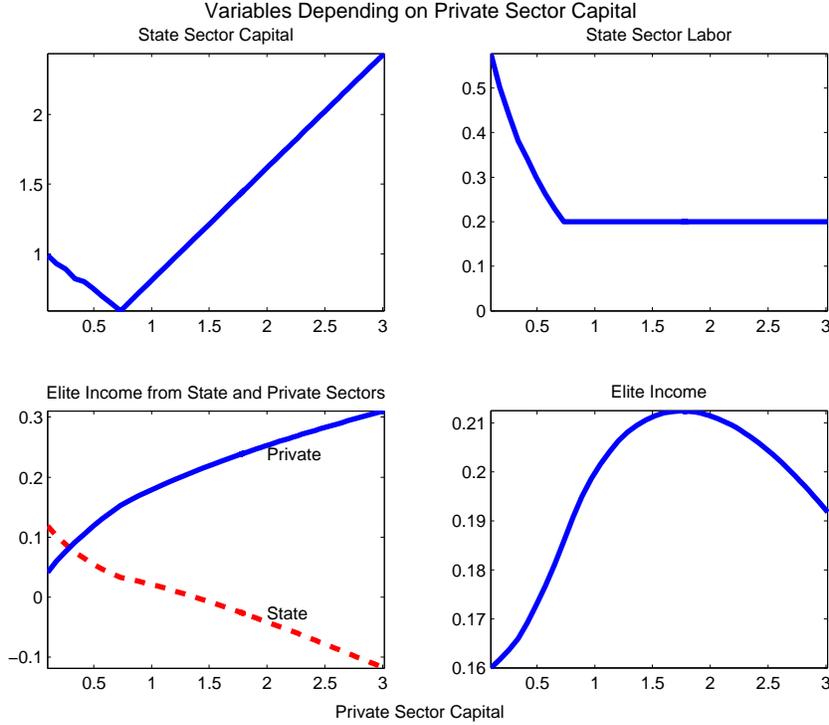


Figure 6: The outcome depending on choice of K_S

capital available for P firms. When the government prefers a larger K_P , it sets $\eta = \bar{\eta}$ and imposes no financial repression. When it wants a smaller K_P , it sets $\eta < \bar{\eta}$, and P firms receive fewer bank loans than the maximal level. This can be seen in figure 7.¹³ The x-axis is a_p . As we move a_p from very small to very large, the P firm leverage goes down gradually (left-upper panel) as the government prefers K_P neither too small or too large. The S sector capital first goes down but then goes up proportionally to the P sector capital (right-upper panel), because enough S employment share needs to be guaranteed (left-lower panel). The government's influence on K_P is limited because η is bounded by $\underline{\eta}$ and $\bar{\eta}$, so it may not be able to set K_P to its preferred level when a_p is too small or too large. This is why the elite lifetime income is maximized for an intermediate level of a_p (right-lower panel). This is the second trade-off for the elite, in addition to the first trade-off of state wage and employment.

Remark 1 (Trade-off of private sector capital). A larger K_P contributes more tax income, but it also requires larger K_S to sustain oligarchy and more interest expense. As K_P increases from a very small level, the elite's current-period income first increases and then decreases. The elite's lifetime income also follows a similar pattern. This trade-off also applies to entrepreneur asset level a_p because it determines K_P .

¹³Figure 7 comes from the same numerical example as figure 5.

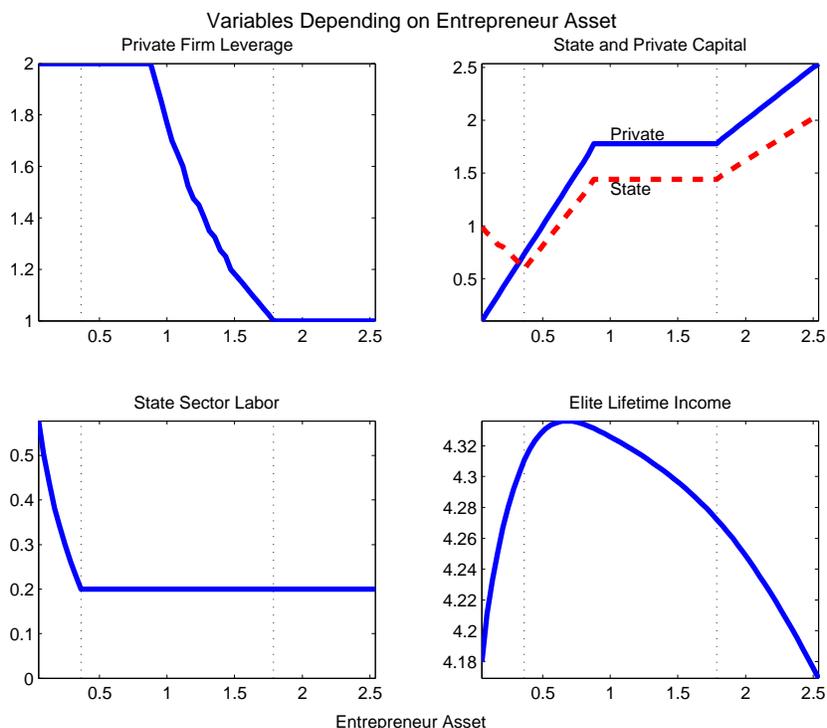


Figure 7: Equilibrium variables, depending on entrepreneurs' asset.

The logic is quite intuitive and illustrates an important capital market trade-off for the elite, in addition to labor market trade-off. The benefit of the private sector to the elite is that it contributes tax. So when the private sector is small, the elite prefers to make it larger and let it contribute more tax, e.g., by lending as much as possible to entrepreneurs, $\eta = \bar{\eta}$. However, the cost for the elite is that private employment can be a threat to the supporter base of the government - state employment. When the private sector capital is large enough, and private sector employs at least $1 - \underline{L}$ workers, the political constraint of the government $L_S \geq \underline{L}$ becomes tight. Then if the elite prefers to stay in oligarchy, it has to invest proportionally in the state sector to maintain the balance between the state and the private sectors. If there is decreasing return to scale and the private sector capital is sufficiently large, the investment in the state sector has to be large and the net return from the state sector can be negative. This is the cost of the private sector for the elite. When the private sector capital is too large, the cost can dominate the benefit and the elite prefers a smaller private sector.

Under which conditions does the government choose to democratize or to sustain oligarchy? The government can invest as much as it wants in S sector to guarantee enough state employment with high wage, for any size of P sector capital. However, large investment in S sector means large cost, while the return can be small due to decreasing return

to scale. If P sector capital is large enough, sustaining oligarchy gives lower lifetime income to the elite compared to democracy - the line for the elite's income in figure (7) can drop below the horizontal zero line: $V(a_p) < 0 = V^D(a_p)$ if a_p is large enough. In this case, the elite chooses to democratize.

3.4.3 Simulation and Dynamics

Given these parameters and the numerical solution, I can simulate the economy starting from a very small private sector: $a_e = 0.05$. Figure 8 and Figure 9 plot key variables and output during the transition in this calibration, where $\underline{L} = 0.2$. If the political system starts in democracy, the transition is the blue dashed line, while the transition in oligarchy is the red solid line. Starting in oligarchy, during the first stage, the private sector is small, and therefore not a threat to oligarchy. The elite encourages the growth of private capital to extract more tax income. So the government sets $\eta = \bar{\eta}$ to lend to private firms as much as possible (left-lower panel of figure 8). Moreover, private firms and entrepreneurs benefit from low wage and abundant labor, so private sector capital grows rapidly (left-upper panel). State employment and capital decline accordingly (right-upper panel). Because the more efficient private sector is reallocated with more capital and labor (right-lower panel), the economic growth is rapid (figure 9). For this reason, this stage is called *rapid growth*.

As the private sector grows larger and the state employment share declines to the critical level \underline{L} , the economy enters the second stage. The declining state employment share threatens the supporter base of oligarchy. If no action is taken, the elite cannot keep their political power any more. So it increases state investment and then restricts private firms' access to the financial market. Because of the policies in favor of state firms, the state sector keeps its relative economic power and the ability to hire \underline{L} labor with high enough wage. The privatization stops, and no more labor reallocation to the more efficient private sector. However, the large investment in state sector can still keep growth high for a while. But the growth gradually slows down because the financial repression on private firms harms the economic efficiency, as shown in the middle section of figure 9. This stage features large state investment and financial repression on private firms, so it is a stage of *state capitalism*. Notice that though the initial output is lower in oligarchy than in democracy, due to the labor market distortion, the output can catch up with democracy in the second stage due to rapid capital accumulation and large state investment.

In the long-run, the elite finds it optimal to always sustain oligarchy. It keeps over-investing in the state sector as the private sector capital grows to its steady state level. Employment share stays at \underline{L} . Though the elite has to pay large investment cost, it still

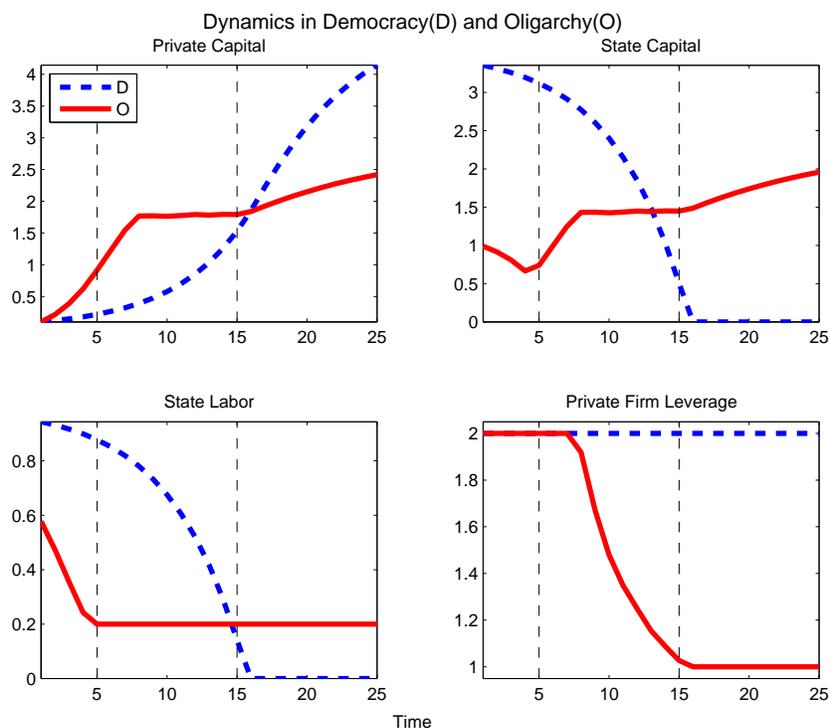


Figure 8: Dynamics in democracy (blue) and oligarchy (red) ending in middle-income trap.

extracts from tax income from the private sector, so it doesn't want to democratize. The economy continues as the second stage: over-investment in state firms, financial repression on private firms, no labor reallocation to private firms. The inefficient capital market harms growth. Furthermore, due to decreasing return to capital, growth gradually slows down and eventually output stops growing at the middle level, which is lower than the level in democracy. So in this case, the third stage is called "*middle-income trap*".

This calibration predicts that China will stay in oligarchy and fall into the middle-income trap, given the current conditions. This is not surprising. The government is right-now strong, politically and economically, meaning that a relatively small fraction of the citizens' support is sufficient to sustain the current regime, and it has enough financial resource - for example, large foreign reserves - to build up the state sector if it needs to. After 2008 financial crisis, the Chinese government initiates the 400 billion stimulus package and bails out mostly state firms while letting many private firms die. This shows that it keeps the economy and resource allocation under control and stable, and it is able to maintain a powerful state sector to guarantee political stability, according to this theory.

Since the political power of the government, captured by \underline{L} is an important parameter determining the cost of sustaining oligarchy and the decision of the elite on whether to

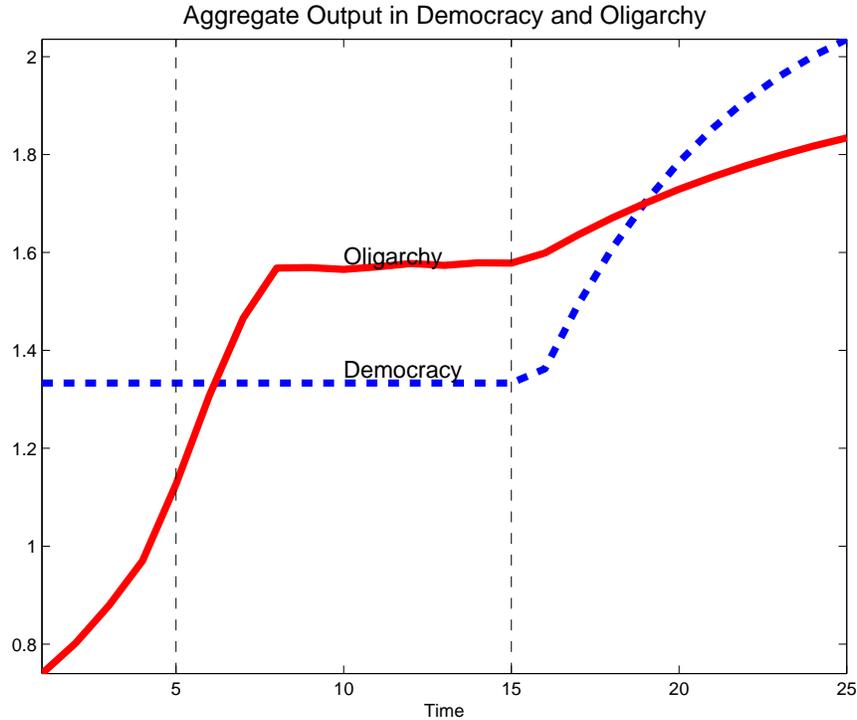


Figure 9: Output in democracy (blue) and oligarchy (red) ending in middle-income trap.

democratize, a large \underline{L} may imply a different long-run development paths. Keeping the other parameters in this calibration, if \underline{L} is changed to be large enough, democratization will occur. In this case, sustaining oligarchy requires many S workers, so the elite has to invest a lot in S sector proportional to the P sector capital. As P sector capital grows larger and larger, the elite finds the cost of maintaining the state sector too large, and it is optimal to democratize for them. This development path is different in the long-run compared to the development in the calibration to China, but it is similar in the early stages: starting from small P sector, in the beginning, P sector employment share grows until it reaches the critical level for sustaining oligarchy; then the government over-invests in S sector to maintain enough supporters for oligarchy; finally the two paths differ in the long-run. This divergence of two paths is the so-called “critical juncture” of development in Acemoglu and Robinson (2012).

The dynamics with $\underline{L} = 0.5$ is simulated and showed figure 10. In this case, the elite chooses to democratize when the private sector capital reaches certain level. The cost for the elite to keep enough workers in the state sector with high wage keeps growing as the private sector capital grows. Additionally, marginal return of capital decreases, so the elite finds the cost of maintaining oligarchy dominates the income in oligarchy when private sector capital grows large enough. It chooses to democratize. As we can see in

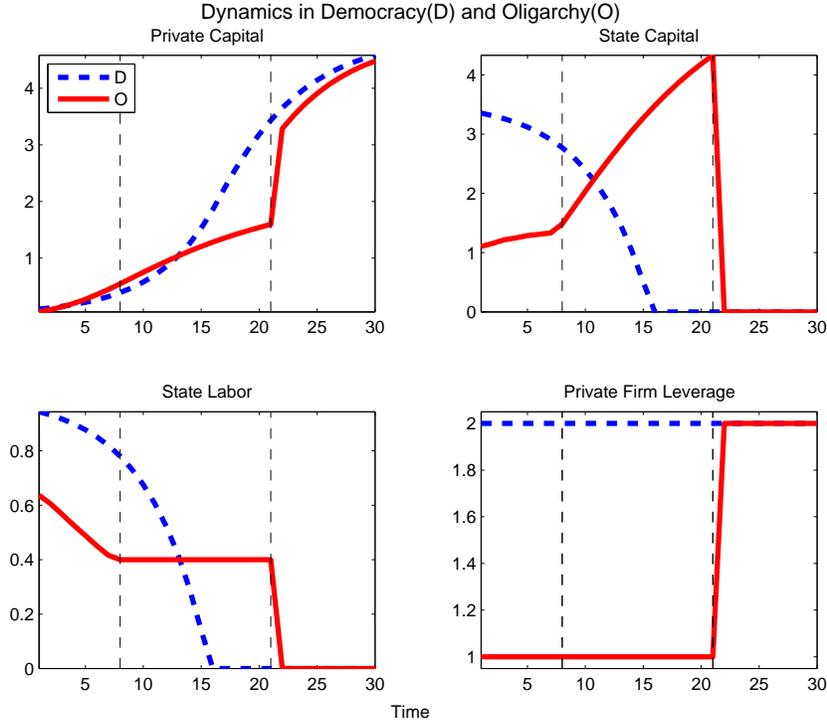


Figure 10: Dynamics in democracy (blue) and oligarchy (red) ending in sustained growth.

figure 10, the state capital quickly drops while the private capital soars up because the financial repression is removed. The output, as shown in figure 11, though slightly goes down due to super rapid decline of the state sector, eventually recovers and converges to the high level in democracy.

In both cases, the transition is featured with three stages, and its properties are summarized in the following.

Remark 2 (Three stage transition). The economy, starting with a small enough private sector, develops along the following path with three stages:

Stage 1: *rapid growth*. Growth rate is high. Private sector grows rapidly, benefiting from the low wage. Moreover, the government encourages private sector growth and does not impose financial repression: $\eta = \bar{\eta}$. Rapid privatization reallocates labor from the state to the private sector.

Stage 2: *state capitalism*. Growth continues. The government over-invest in the state sector, while restricting private firms' access to the financial market: $\eta < \bar{\eta}$. Privatization stops and the state employment share stays at the critical level \underline{L} .

Stage 3: Two cases.

Case 1: *middle-income trap*. Oligarchy is sustained permanently and growth slows down. State investment keeps growing at the same rate of the private sector capital, to keep

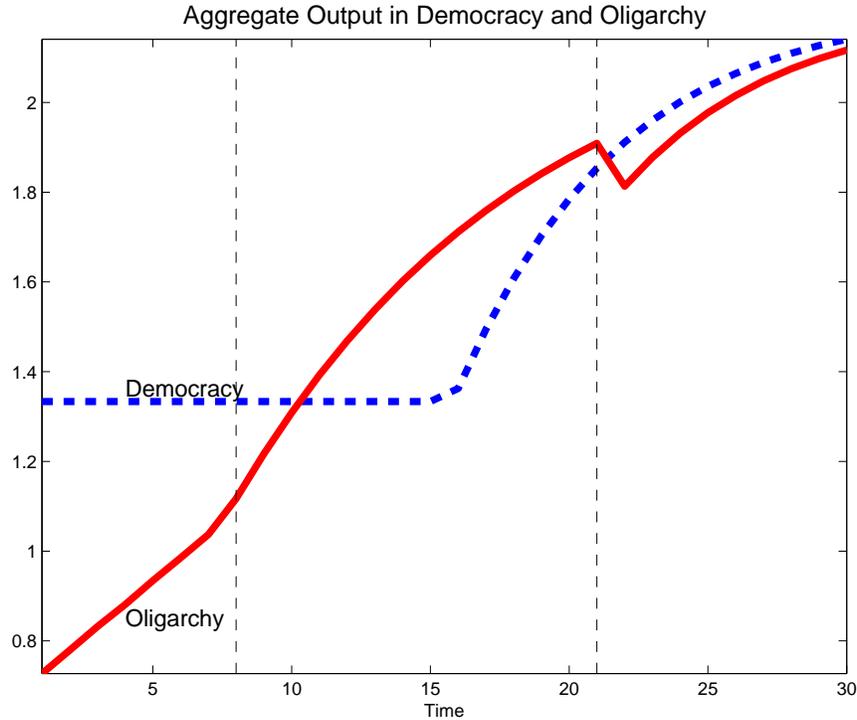


Figure 11: Output in democracy (blue) and oligarchy (red) ending in sustained growth.

state employment share at \underline{L} . Financial repression on private firm reaches the tightest level $\eta = \underline{\eta}$. This happens if \underline{L} is sufficiently small.

Case 2: *sustained growth*. Democratization occurs and output growth becomes rapid again. Financial repression and labor market distortion disappear. State sector declines while private sector grows. This happens if \underline{L} is sufficiently large.

3.5 Discussions and Policy Implications

3.5.1 Implications of the Equilibrium Given Capital Allocation

Given capital allocation, the government creates a dual labor market: state workers get high wages and hence support the government, while private workers get low wages. This is the so-called “divide-and-rule” strategy: breaking workers into two sub-groups, and providing different economic interests to gain support from one group at the cost of the other.

This equilibrium given capital allocation is consistent with three phenomena in current China: (1) large state-private sector wage gap, (2) middle class’s political support for the current regime, and (3) higher capital labor ratio and low capital return in the state sector.

The first fact is discussed in section 2, and is the immediate consequence of proposition 1. High state sector wages are necessary for getting political support from workers, and the general equilibrium effect leads to abundant and cheap labor in the private sector. Entrepreneurs and private firms benefit from the abundant cheap labor, in the short-run. This allows potentially faster capital accumulation and growth of the private sector and the whole economy. We will discuss more on this in the dynamic model.

Second, the middle class workers, state workers in the model, are supportive to the existing political regime because of the economic benefits they receive.¹⁴ This is consistent with the finding of Chen and Lu (2011) discussed in section 2, but contrary to the traditional wisdom that the middle class are the natural driving force of democracy, as in the European history. In the history of democratic movements in Europe, such as the Glorious Revolution and French Revolution, the middle class was against the aristocracy of the kings, whose political power relied on repression. The middle class did not rely on the state but emerged from private businesses. In contemporary China, the state sector is large and a significant fraction of the middle class have been created by and rely on the state, and in turn become supporters of the state. It is also similar in many other developing countries. This helps to understand why in some emerging markets, economic growth and the burgeoning bourgeoisie do not automatically lead to demand for democratization. For example, as reported in *The Economist* 2009, 95% of adult Kuwaitis work for the government, usually in white-collar civil-service jobs which are typical middle class jobs, while its private-sector middle class consists almost entirely of foreigners. The wage gap between the state and private sector is large there. These distortions keep politically important local workers in the state sector and is an efficient way to maintain oligarchy.

The third fact is well documented in the literature. Song et al. (2011) show that state sector capital labor ratio is much larger than the private sector, in every industry. Brandt and Zhu (2010) show that the capital return in the state sector is lower than 5% while the number for the private sector is above 50%. The difference in capital returns is partly due to the difference of wages and distorted labor allocations. The other reason is capital allocation, as we will see in the dynamic model below.

In a nutshell, the above analysis on the equilibrium given capital allocation is useful to illustrate the interactions of the political and economic systems in oligarchy in each period. On the one hand, the political interests, largely shape the state distortions and economic outcome. Taking into account political considerations, we can explain many

¹⁴Entrepreneurs, as the other group of the middle class in the model, also support oligarchy. Their short-run income is higher in oligarchy, as is their lifetime income in most cases in the calibrated dynamic model.

economic phenomena and puzzles in China. On the other hand, economic power determines political outcome. Only when the state sector is economically powerful and equipped with enough capital, can the elite keep a large enough supporter base to sustain oligarchy.

3.5.2 Discussion of the Dynamic Equilibrium

The first two stages of the calibrated dynamic model are consistent with China's recent development. From 1997 to around 2003, it is a stage of rapid privatization, as the state employment share declines dramatically. The private sector, in terms of employment share and GDP, grows rapidly, for two reasons. First, the wage is low in the private sector. Compared to state firms which face the regulations on the wage and other payments, including pension tax, health insurance, unemployment insurance and so on, private firms pay relatively low wages, which result in high capital returns. Therefore, private firms accumulate capital rapidly and grow fast. The low wage keeps Chinese private firms competitive. It contributes a lot to the growth of export, and the growth of the economy. Second, the government encourages the private sector growth, because a larger private sector contributes more tax while it is still not too large to threaten the supporter base of the government - state employment. So the government encourages various financial resource flowing into the private sector, not only bank loans but also foreign direct investment (FDI), and so on.

At around 2003, as the state employment share approaches the critical level, the privatization slows down and stops dramatically. The direct reason is that more and more investment is diverted to state firms but not private firms. State sector investment share stays at around 60% though its employment is much smaller (see Brandt and Zhu (2010)). The state over-investment retains state employment, but reduces the capital return. In the private sector, the capital return is high, not only because they are more efficient, but also because of the credit constraint: private firms cannot get enough bank loans for their high return projects. In fact, the financial constraint on private firms has been getting tighter over time, signaling growing financial repression on them. The growing financial repression on private firms is formally documented as the growing state-private capital wedge in Brandt and Zhu (2010). The protection on state firms and repression on private firms have gained much attention and are called "the state advances as the private sector retreats". For example, in the passenger airline industry, by 2006, eight private carriers had grown rapidly and had challenged the three state-controlled majors, thanks to the previous government policies encouraging private investors to enter. However, afterward, the government starts supporting the state airlines and keeps them alive with policies in-

cluding stock purchase from the central government. The state airlines not only survived and also are able to keep their dominance. Our theory's prediction indeed explains why this is happening in the second stage "state capitalism". The elite prefers to maintain a sufficiently strong state sector to guarantee the political control. This model's prediction on the capital return in the second stage is broadly consistent with the trend: a large gap between the state and private capital returns and declining state capital return. Though the capital return in state firms is so slow, the government still keep investing into them to keep them alive.

3.5.3 Reforms

Is China doomed to fall into the middle-income trap? Not necessary. If the underlining conditions change, the policies and the development path can change accordingly. Mapping into the model, if the parameters such as $\underline{L}, \underline{\eta}, z_S$ change, the government policies and the dynamics, including the third stage, will change. Many policy suggestions on how to switch China's development to a more sustainable path have been made by economists and China watchers. For example, Gary Becker suggested that financial reform should be taken to allocation more resource to private firms, and rural immigrants should be given more rights. Will the government take the suggestions and implement all the policies and reforms that sustain growth? We need to notice that policies or reforms that benefit economic growth may not benefit the elite, who is very influential in the government.

Suppose the government takes a reform that gives more political rights to workers, especially the immigrant workers. This implies that the government has to buy support from a larger fraction of the population. We know that if \underline{L} increases from 0.2 to 0.5 leads to democratization and sustained growth. But does the elite like that? Its income goes down to 0 if democratization occurs, so obviously this reform will encounter strong resistance from the elite.

In the above model, we assume that the government is completely under the control of the elite. Some may believe that, in some cases, some technocrats become powerful in the government, and they care only about the output growth in the long-run, but not the economic benefit of the elite. In this case, they can initiate reforms which correspond to changing the key parameters of the model, such as $\underline{L}, \underline{\eta}, z_S$. To which extend they can push the reform to depends on their political power in the government. P can be one of the key parameters $\underline{L}, \underline{\eta}, z_S$. Notice that we consider reform as changing parameters but not the endogenous policy variables such as K_S, η . This implies that technocrats only get a key moment to push for a big change of the society and the political and economic system, and afterwards, the government decisions will be made by the elite.

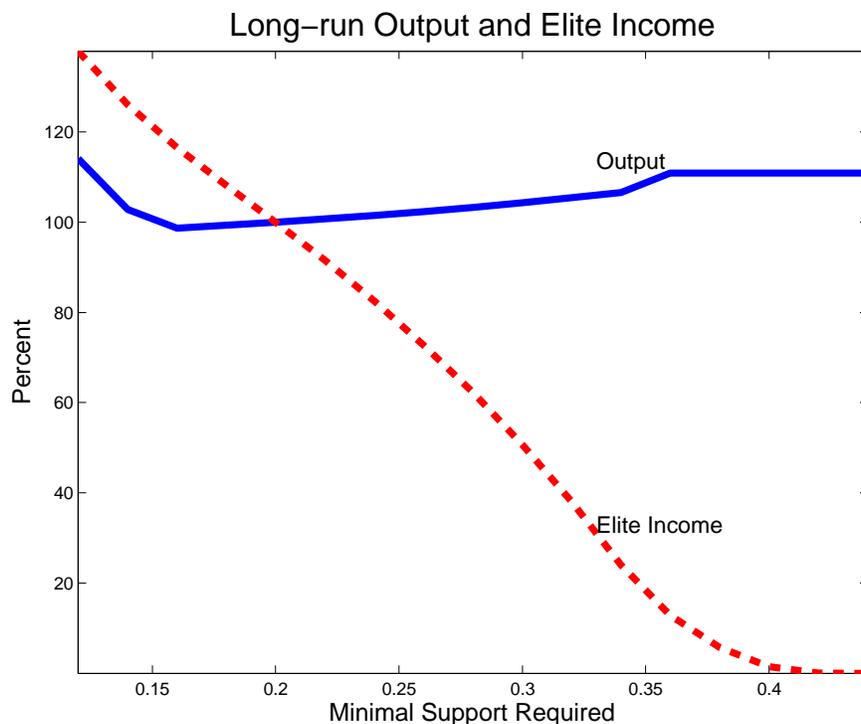


Figure 12: elite income and long-run output depending on \underline{L} .

Figure 12 shows how the elite's lifetime income V , and the long-run output Y_∞ which the technocrats care respond to different levels of \underline{L} . Technocrats would like to increase workers' political rights and increase \underline{L} from the current level $\underline{L} = 0.2$, because this makes the government invest more in the state sector, or even choose to democratize. Both of them lead to larger output levels. However, the reform as the result of the bargaining can only push \underline{L} to the right limited by α . If α is small, the increase of workers' political rights won't be large.

Similarly, financial reform, which reduces the financial repression on private firms can be considered as increasing $\underline{\eta}$. It again increases output, because the private firms can grow larger, and it may even leads to democratization. But again, it harms the elite interests and is hard to be implemented.

One exception is the state firm reform. If a successful reform is taken to increase state firm productivity and reduce the TFP gap between the private and state firms, it increases the output potential. More than that, under the condition that oligarchy is sustained, a more efficient state sector implies that the government can allow the private sector to grow more without worrying about their supporter base - state workers. Less financial repression on private firm is needed and higher economic efficiency can be achieved. This reform also increases elite income because of higher total output. Figure 13 plots how the

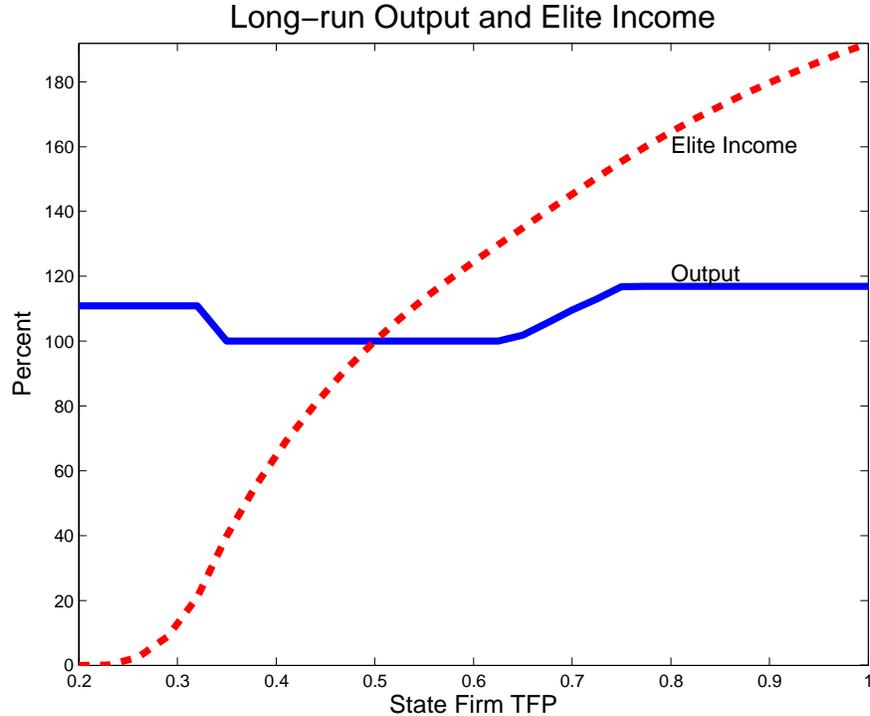


Figure 13: elite income and long-run output depending on z_S .

long-run efficiency, measured as the long-run output in oligarchy over democracy, can be improved by a more efficient state sector (in the region $z_S \in [0.6, 0.75]$), while the elite income always increase with that. This reform is more likely to be implemented the government. In fact, this is happening right now in China. Hsieh and Song (2013b) document the state-private TFP gap is declining. The so-called “industrial upgrading”, which aims at building high-tech state firms, is at the top of the agenda for China’s further economic reforms. However, it is also very difficult to completely close the gap between the state and private firms, because they are less flexible and provide less economic incentives for the managers, compared to private firms.

4 Conclusion

This paper provides a political-economic theory to study China’s future economic and political transition and to understand China’s recent development. Based on a dynamic growth model, I add the political constraint that the ruling elite faces: sufficient political supporters. To satisfy the constraint, the government creates a dual labor market, which gives high wages to state workers to turn them into supporters. Moreover, in the financial market, the government encourages private sector growth when it is small enough,

but switches to protecting state sector and restricting the private sector when the private sector capital is too large. The economic policies lead to a three-stage transition. The first two stages are *rapid growth* and *state capitalism*, which are consistent with several salient aspects of China's development, including (1) rapid growth with repressed wage in the private sector; (2) political support from the middle class, including state sector workers and private entrepreneurs; and (3) financial constraints on private firms and support for state firms. In the future, i.e., the third stage of development, China is likely to enter a *middle-income trap* given the current conditions, especially the economically and politically powerful state. To switch to the other development path that leads to *sustained growth*, necessary reforms have to be taken, though such reforms may face resistance from the elite.

Even though the focus of this paper is on China, it is also useful to study the development of many other emerging countries and even some developed countries with similar patterns compared to China. First, the key political constraint in the theory also exist in some other countries such as Kuwait, Korea in the 1980s, as the political elite or politicians need to buy political support from public workers or workers in industries under their control. Similar stories have occurred in these countries. Before the 1990s, the large local conglomerates (chaebol) in Korea were granted privileged access to low-cost credit. In Kuwait, the oil industry is under the control of the government, so the public sector can hire more than 90% of Kuwaiti nationals with relatively high wage while the private sector is populated with expatriates. Second, the theory is also useful to think on a question in development, namely, whether other developing countries should apply the "China model", i.e., the combination of authoritarian politics and state-guided capitalism, to promote economic growth. Some suggestions in favor of adopting this model are based on its past success, but the long-run outcome should be carefully examined and distinguished from the short-run effect. This theory provides a quantitative framework to evaluate the economic and political consequences.

Further empirical work can be done to examine the theory, especially the three-stage political-economic transition. Which conditions determine the transition to democracy and long-run growth? Is it consistent with the theory? The theory predicts that if a country can easily build a large state sector, for instance due to rich natural resource, is more likely to sustain the oligarchy, while if efficiency is very important for a country, for example because of exposure of international competition, democratization is more likely to occur. Anecdotal evidence about Gulf countries compared to export oriented economies like Taiwan seem to support the theory. Still, more systematical evidence will be useful to check and improve the theory.

References

- Acemoglu, D. (2003). Why Not a Political Coase Theorem? Social Conflict, Commitment, and Politics. *Journal of Comparative Economics*, 31(4):620–652.
- Acemoglu, D. (2008). Oligarchic Versus Democratic Societies. *Journal of European Economic Association*, 6(March):1–44.
- Acemoglu, D., Aghion, P., and Zilibotti, F. (2006). Distance to Frontier, Selection, and Economic Growth. *Journal of European Economic Association*, 4(March):37–74.
- Acemoglu, D., Egorov, G., and Sonin, K. (2008). Coalition Formation in Non-Democracies. *Review of Economic Studies*, pages 987–1009.
- Acemoglu, D., Egorov, G., and Sonin, K. (2012). Dynamics and Stability of Constitutions, Coalitions, and Clubs. *American Economic Review*, 102(4):1446–1476.
- Acemoglu, D. and Robinson, J. (2005). *Economic Origins of Dictatorship and Democracy*.
- Acemoglu, D. and Robinson, J. (2012). *Why Nations Fail: Origins of Power, Poverty and Prosperity*.
- Bai, C., Hsieh, C.-t., and Qian, Y. (2006). The Return to Capital in China. *Brookings Papers on Economic Activity*, pages 61–88.
- Benjamin, D., Brandt, L., Giles, J., Wang, S., and Behrman, J. (2005). Income Inequality During China’s Economic Transition.
- Besley, T. and Persson, T. (2009). The Origins of State Capacity: Property Rights, Taxation, and Politics. *American Economic Review*, 99(4):1218–1244.
- Besley, T. and Persson, T. (2011). *Pillars of Prosperity*.
- Brandt, L., Tombe, T., and Zhu, X. (2012). Factor Market Distortions Across Time, Space and Sectors in China. *Review of Economic Dynamics*, 16(1):39–58.
- Brandt, L. and Zhu, X. (2000). Redistribution in a Decentralized Economy growth and Inflation in China under Reform. *Journal of Political Economy*, 108(2):422.
- Brandt, L. and Zhu, X. (2010). Accounting for China’s Growth.
- Chen, J. and Lu, C. (2011). Democratization and the Middle Class in China: The Middle Class’s Attitudes toward Democracy. *Political Research Quarterly*, 64(3):705–719.

- Eichengreen, B., Park, D., and Shin, K. (2013). Growth Slowdowns Redux: New Evidence on the Middle-income Trap.
- Fatás, A. and Mihov, I. (2009). Another Challenge to China's Growth. *Harvard Business Review*.
- Ge, S. and Yang, D. T. (2014). Changes in china's wage structure. *Journal of the European Economic Association*, 12:300–336.
- Gill, I. and Kharas, H. (2007). *An East Asian Renaissance: Ideas for Economic Growth*.
- Giordano, R., Depalo, D., Pereira, C., Eugène, B., Papapetrou, E., Perez, J. J., Reiss, L., Pereira, M. C., and Roter, M. (2011). The Public Sector Pay Gap in a Selection of Euro Area Countries. *European Central Bank Working Papers*.
- Han, L. and Kung, J. K.-s. (2012). Fiscal Incentives and Policy Choices of Local Governments: Evidence from China.
- Hsieh, C.-t. and Klenow, P. J. (2009). Misallocation and Manufacturing TFP in China and India. *Quarterly Journal of Economics*, CXXIV(November):1403–1448.
- Hsieh, C.-t. and Song, Z. (2013a). Grasp the Large , Let Go of the Small : The State Sector in China.
- Hsieh, C.-t. and Song, Z. (2013b). Grasp the Large, Let Go of the Small: the Transformation of the State Sector in China.
- Huang, Y. (2008). *Capitalism with Chinese Characteristics: Entrepreneurship and the State*.
- Lewis, W. A. (1954). Economic Development with Unlimited Supplies of Labour. *Manchester School*, pages 139–91.
- Lindbeck, A. (1995). Hazardous Welfare-State Dynamics. *American Economic Review, Papers and Proceedings*.
- Lipset, S. M. (1959a). Some Social Requisites of Democracy: Economic Development and Political Legitimacy.
- Lipset, S. M. (1959b). Some Social Requisites of Democracy: Economic Development and Political Legitimacy.
- Melly, B. (2002). Public-private Sector Wage Differentials in Germany: Evidence from Quantile Regression. *Empirical Economics*, 30(2):1–26.

- Moll, B. (2014). Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation? *American Economic Review*, pages 3186–3221.
- Mueller, R. E. (1998). Public-private Sector Wage Differentials in Canada: Evidence from Quantile Regressions. *Economics Letters*, 60(2):229–235.
- Musacchio, A. and Lazzarini, S. G. (2012). Leviathan in Business : Varieties of State Capitalism and their Implications for Economic Performance. (June 2012):1–63.
- OECD (2013). *Government at a Glance*. Government at a Glance. OECD Publishing.
- Poterba, J. M. and Rueben, K. S. (1994). The Distribution of Public Sector Wage Premia: New Evidence Using Quantile Regression Methods. *NBER Working Paper*, (May).
- Reinikka, R. and Svensson, J. (2004). Local Capture: Evidence from a Central Government Transfer Program in Uganda. *Quarterly Journal of Economics*, (May).
- Robertson, P. E. and Ye, L. (2013). On the Existence of a Middle Income Trap.
- Robinson, J. A. and Acemoglu, D. (2001). Inefficient Redistribution. *American Political Science Review*, pages 649–661.
- Robinson, J. a. and Torvik, R. (2005). White elephants. *Journal of Public Economics*, 89(2-3):197–210.
- Robinson, J. A. and Verdier, T. (2003). The Political Economy of Clientelism. 115(2):260–291.
- Robinson, J. A. and Verdier, T. (2013). The Political Economy of Clientelism. 115(2):260–291.
- Sokoloff, K. L. (2002). De Facto Political Power and Institutional Persistence.
- Song, Z., Storesletten, K., and Zilibotti, F. (2011). Growing Like China. *American Economic Review*, 101(February):196–233.
- Storesletten, K. and Zilibotti, F. (2014). China’s Great Convergence and Beyond. *Annual Economic Review*.
- Tsai, K. (2007). *Capitalism without Democracy: The Private Sector in Contemporary China*.
- Xia, Q., Song, L., Li, S., and Appleton, S. (2011). Effect of Public Sector and SOEs on Wage Inequality of Urban China. pages 1988–2007.

5 Appendix

5.1 Proof of Proposition 1

In the main text, we show that given capital, high enough wage to buy support from S workers implies $L_S \leq \bar{L}$. Meanwhile, enough supporters means $L_S \geq \underline{L}$. If $\bar{L} \geq \underline{L}$, or equivalently, $v \frac{zK_S}{zK_S + K_P} \geq \underline{L} \Leftrightarrow \frac{K_S}{K_P} \geq \frac{z\underline{L}}{v - \underline{L}}$, the political constraint can be satisfied by setting w_S such that $L_S \in [\underline{L}, \bar{L}]$, and oligarchy can be sustained.

If oligarchy is sustained, given that $w_S \geq y_w^D > w_P$, shown in the main text, we know that the wage is higher in S sector than in P sector. Moreover,

$$\begin{aligned} (1 - \alpha) (z_S K_S)^\alpha L_S^{-\alpha} &> (1 - \alpha) (K_P)^\alpha L_P^{-\alpha} \Rightarrow \\ \frac{K_S}{L_S} &> \frac{1}{z_S} \frac{K_P}{L_P} > \frac{K_P}{L_P}, \end{aligned}$$

which states that the capital labor ratio is higher in S sector. Given the larger capital labor ratio in S sector, capital return is obviously lower:

$$\begin{aligned} \alpha z_S^\alpha K_S^{\alpha-1} L_S^{1-\alpha} &< \alpha K_S^{\alpha-1} L_S^{1-\alpha} \\ &< \alpha K_P^{\alpha-1} L_P^{1-\alpha}. \end{aligned}$$

Compared to in democracy, entrepreneurs get cheap and abundant labor in P sector. As shown in the main text, $L_P > L_P^D$ and this implies higher P sector capital return

$$\alpha K_P^{\alpha-1} L_P^{1-\alpha} > \alpha K_P^{\alpha-1} (L_P^D)^{1-\alpha},$$

and higher entrepreneur income, which is simply the after-tax capital return, minus depreciation.

5.2 Proof of Proposition 2

In democracy, if S firms still exist, each elite member supplies capital to S sector competitively and she can borrow freely at the interest rate r . This implies that the return to the elite's capital supply to S sector has to be r . If the return to the elite is greater than r , S sector capital supply becomes positive infinity. If the return is lower than r , no elite supplies capital. In the case that S sector does not exist at all, the elite can simply save the asset in the bank the gets the return r . In both cases, the return in each period is r . An elite member in democracy cannot tax others and gets no transfer, so she only replies on

asset return and her income from other sources is simply 0.

An entrepreneur, supplies capital to P sector by using her own asset and borrowing from the bank, and get return. Suppose that the net return of P sector capital to entrepreneurs - after paying tax and depreciation - is r_{pt} . Then obviously, if $r_{pt} > r$, the representative entrepreneur is willing to borrow as much as possible and supply as much as possible capital to P sector: $K_{pt} = \eta_t a_{pt}$, and the return to her asset is $r_{pt} = \frac{r_{pt} \eta_t a_{pt} - r(\eta_t - 1)a_{pt}}{a_{pt}} = r + (r_{pt} - r) \eta_t > r$. If $r_{pt} \leq r$, the return to an entrepreneur's asset is simply r , as she can at least save in the bank and get the return r . An entrepreneur takes return to P sector capital and leverage η as given, and the return to her asset in each period r_p is also determined. An entrepreneur standing in period 0 holding asset a_{p0} solves:

$$\begin{aligned} & \max_{\{c_{pt}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_{pt} \\ \text{s.t. } & \sum_{t=0}^{\infty} \frac{c_{pt}}{\prod_{t'=0}^t (1 + r_{pt'})} \leq (1 + r_{p0}) a_{p0}. \end{aligned}$$

Given the property of log-utility, it is easy to see that the solution of this problem is simply

$$\begin{aligned} c_{pt} &= (1 - \beta) (1 + r_{pt}) a_{pt}, \\ a_{pt+1} &= \beta (1 + r_{pt}) a_{pt}. \end{aligned}$$

If $\beta > (1 + r)$, and given that $r_{pt} \geq r$, a_{pt} keeps growing.

In democracy, if S firms still exist, competition of S sector capital supply implies

$$r_S^D = (1 - \tau^D) \alpha z_S^\alpha K_S^{\alpha-1} (L_S^D)^{1-\alpha} - \delta = r.$$

This determines S sector capital labor ratio and wage:

$$\begin{aligned} \frac{K_S}{L_S^D} &= \left(\frac{r + \delta}{(1 - \tau^D) \alpha z_S^\alpha} \right)^{\frac{1}{\alpha-1}} \Rightarrow \\ w^D &= (1 - \alpha) \left(\frac{z_S K_S}{L_S^D} \right)^\alpha. \end{aligned}$$

The wage pins down the private sector labor, given capital:

$$w^D = (1 - \alpha) \left(\frac{K_P}{L_P^D} \right)^\alpha \Rightarrow$$

$$L_P^D = \left(\frac{w^D}{1 - \alpha} \right)^{-\frac{1}{\alpha}} K_P.$$

When S firms still exist, obviously P sector capital return after paying tax and depreciation is higher than the counterpart in S sector, which is r . Then $K_P = \bar{\eta} a_p$, and if a_p keeps growing, L_P^D keeps growing and L_S^D declines gradually. When a_p reaches $\frac{1}{\bar{\eta}} \left(\frac{w^D}{1 - \alpha} \right)^{\frac{1}{\alpha}}$, L_P^D reaches 1, and S sector becomes 0. Afterwards, entrepreneurs keep accumulating assets, and the economy behave like a neo-classic growth model.

5.3 Proof of Lemma 1

This is in fact already proved in the proof of proposition 2. Entrepreneurs take the return to capital in P sector and leverage η as given in both democracy and oligarchy, so their behavior are basically the same in both regimes.

5.4 Proof of Lemma 3

Denote the lifetime utility achieved by solving the two sub-problems - first maximizing lifetime income and then maximizing lifetime utility - as U^O .

First, $U^O \leq W^O$. Denote the solution for V^O as $\{\hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t, \hat{i}_t\}_{t=0}^\infty$, where $\hat{i}_t \in \{O, D\}$ is the choice of staying in oligarchy or not. The corresponding consumption and saving decisions obtaining U^O is denoted as $\{\hat{c}_{et}, \hat{a}_{et+1}\}_{t=0}^\infty$. Combine them together, the choice $\{\hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t, \hat{i}_t, \hat{c}_{et}, \hat{a}_{et+1}\}_{t=0}^\infty$ is a feasible choice of the original problem. This is because in each period, for any level of a_e , the choice sets of w_S, K_S, η in the original problem are the same.

Second, $U^O \geq W^O$. Consider the choice that solves the original problem and achieves W^O and denote it as $\{w_{St}^*, K_{St}^*, \eta_t^*, i_t^*, c_{et}^*, a_{et}^*\}_{t=0}^\infty$. Compare $\{w_{St}^*, K_{St}^*, \eta_t^*, i_t^*\}_{t=0}^\infty$ with the solution of V^O - $\{\hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t, \hat{i}_t\}$, one can see that $\hat{V}^O = \sum \frac{y_{et}}{R^t} > V^{O*} = \sum \frac{y_{et}^*}{R^t}$. Then in the second sub-problem, choosing $\hat{c}_{e0} = c_{e0}^* + \hat{V}^O - V^{O*}$, $\{\hat{c}_{et}, \hat{a}_{et}\}_{t=1}^\infty = \{c_{et}^*, a_{et}^*\}_{t=1}^\infty$ is feasible and gives at least as high lifetime utility as $\{w_{St}^*, K_{St}^*, \eta_t^*, i_t^*, c_{et}^*, a_{et}^*\}_{t=0}^\infty$.

To sum up, $U^O = W^O$. Solving the original lifetime utility maximization problem is the same as solving the two sub-problems.