Who pays for job training?

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June 21, 2012

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Abstract

A puzzling feature of the UK labour market is that there is not enough investment in job training (either by workers or by firms) while there is a high skill premium. We model this as a two sector (skilled and unskilled) economy with a non-cooperative training game between vacant skilled firms and unemployed unskilled workers. A vacant skilled firm has an incentive to train an unskilled worker because of the chance of a better match with a skilled worker. On the other hand, an unskilled worker has an incentive to train because it could increase his lifetime earning. Using a social planning problem as a baseline, the paper demonstrates that while it is socially optimal to invest in job training, the private sector may fail to internalize these benefits in a wide range of economies. In a Nash equilibrium, the training propensity of firms is higher if they have greater bargaining strength while workers tend to invest less in training if they have weak bargaining strength. Calibrating the model for the UK economy, we find that the welfare gain from a training programme is highest if workers instead of firms bear the cost of training. The model also predicts that while the skill gap decreases, the income inequality could rise when a job training programme is in place.

KeyWords: Education; UK Labor Market; Calibration

JEL Codes: J24, J44, J63

1　Introduction

A striking feature of the UK labour market is the simultaneous presence of a staggering skill premium between high and low skilled jobs and underinvestment in job training. Since 1986, more jobs require advanced skills. According to Felstead et al. (2002), a substantial skill premium exists at the graduate level (57% for women and 38% for men) compared with jobs which require no qualification. Felstead et al. (2002) also document that there is a shortage of high-skilled workers (level 4) while there is an excess supply of workers with intermediate skills and low skills (levels 3 and 2).[1]

While there is a high skill premium, there is little evidence of worker participation in training. Schömann and Siarov (2005) provide evidence of low worker participation in training in the European union. Although there was a steady increase in training participation (about 2.5%) over the period 1995-2003, it significantly falls short of the Lisbon target of 15%. They also observe a remarkable disparity in training participation of low and high skilled workers. The participation rate of low skilled workers in a training programme is 13% less than high skilled workers.

These findings are puzzling. If there is such a high skill premium, why is this not exploited by the workers and firms? If a large number of skilled, well paid jobs are vacant, one expects that workers would invest in job training and reap the benefits. Instead, workers are content to acquire lesser intermediate qualifications in the labour market. Low skilled workers hardly undertake any investment in training to upgrade their skills. There is also little evidence that skilled firms train unskilled workers while skill shortages are persistent. The recent interim review of the UK Commission for Employment and Skills (2010) points out the importance of demand and supply of high skilled workers in the UK economy. This survey raises the question how demand and supply of skilled workers could be an interdependent phenomenon. An economy could be trapped in a low level equilibrium with low demand for skilled workers as well as a low supply simply because there may not be enough incentive for investment in skill enhancement.[2] The bottom-line is that a high skill premium coexists with underinvestment in training.

The aim of this paper is to understand the reasons for this voluntary underinvestment in training

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[1] The following quotation from Felstead et al. (2002) aptly summarizes this supply-demand imbalance: "...there are 6.4 million people qualified to the equivalent of NVQ level 3 in the workforce, but only 4 million jobs that demand this level of highest qualification. There are a further 5.3 million people qualified at level 2, but only 3.9 million jobs that require a highest qualification at this lower level".

[2] Haskel and Martin (2001) document that the skill shortage is greater for firms employing advanced technology.

and to quantify the cost of training. A long standing literature exists on the financing of training dating back to the seminal work of Becker (1965). Becker pointed out that in a competitive labour market environment firms have no incentive to provide general training to workers as this training is fully transferrable to other firms. There is disagreement in the subsequent literature whether firms would benefit from training workers. Acemoglu and Pischke (1999) and Stevens (1994) argue that when firms have labour market power, they may reap some benefits from investing in general training.

Our paper departs from the existing literature in several important ways. First, training is a binary decision, either train or do not train. We consider two kinds of training possibilities, namely unskilled workers' self training and vacant skilled firms imparting on-site training to the worker. The cost is borne by the agent who takes the decision. This is an important departure from Acemoglu and Pischke (1999), who assume that only firms train workers after a matching consummates and then the cost of training is split between workers and firms. The fact that there are two kinds training possibilities gives rise to a hold-up problem in our setting. Either worker or firm may just wait for the other party to invest in training. No investment in training may thus arise as a Nash equilibrium.

The second important departure is that in our model the decisions by workers and firms regarding training changes the probability of matching which means that the likelihood of matching is endogenous. This has profound implications for the wage structure; skilled wages are higher if firms bear the cost of training. The reason is that firms incur a sunk cost by training an unskilled worker. To retain the trained worker, the firm pays a higher efficiency wage. This raises the skilled wage generally when firms train workers.

Third, we ask when training is socially optimal but not privately optimal. We answer this question by explicitly setting up a social planning problem where the social planner internalizes all the search and vacancy costs. Since firms and workers could fail to internalize some of these costs, a state of no-training could arise as a Nash equilibrium which may not be socially optimal. Using a calibrated version of the model, we estimate the welfare gain from training using our social planning model as a baseline.

Our key theoretical result is that the propensity of worker's or firm's involvement in the training programme (determined by a threshold schooling cost at which the worker or firm is indifferent between training and no training) depends on the relative bargaining strength of the firms vis-a-vis workers. If workers have low bargaining strengths they will be reluctant to bear the cost of self training because

even after going through such a costly training programme, firms may not remunerate them well enough. On the other hand, if firms have strong bargaining power, they will be more willing to bear the cost of training. This is a result that echoes Acemoglu and Pischke (1999) although Acemoglu and Pischke do not allow the possibility of worker's self training and the issue of a hold-up problem in training.

Our calibrated model for the UK predicts that a firm's propensity to provide onsite training is rather low. This reflects the poor bargaining position of the firms vis-a-vis workers. In fact, the overall welfare gain is higher if instead of firms, unskilled workers bear the cost of training. For our calibrated economy the overall welfare gain from job training can range from 7 to 10%. Skilled firms can lose about 23% of welfare when they invest in training while they will gain by 11% if instead of them, workers invest in training. The overall gain in welfare for all parties is 4% higher when instead of firms, workers bear the cost of training. The welfare gain could be higher if skilled firms are subsidized by the government or the unemployment benefit is reduced for the workers making leisure expensive to them. We also find that the income inequality could rise when a job training programme is in place. This happens primarily because a job training programme increases the skilled unemployment in our model because the number of skilled firms do not change but the number of skilled workers increase.

The paper is organized as follows. In the following section, we lay out the environment and set up a model of strategic job training. Section 3 describes the social planning model. Section 4 reports the quantitative analysis of the model. Section 5 concludes.

2 The Model

Two types of technologies are available: high-skill (suffixed as s) and low-skill (suffixed as u). Each of these technologies could produce p^s and p^u units of output if it is operated by a skilled and unskilled worker respectively. The skilled sector has a higher productivity which means $p^s > p^u$. There are continuum of such skilled and unskilled workers and firms in a unit interval. Initially there are μ_0^{sw} proportion of skilled workers and μ_0^{sf} firms. There is also an initial distribution of vacant skilled and unskilled firms denoted as v_{00}^s and v_{00}^u respectively and an initial distribution of unemployed skilled and unskilled workers denoted as u_{00}^s and u_{00}^u respectively.

There are two types of provisions for job training in the economy: (i) unskilled unemployed worker

undertakes self-training by joining a skill center, and (ii) vacant skilled firm imparts job training to an unskilled worker. In both cases, an unskilled worker turns skilled in the next period. The only decision problem for either the unskilled unemployed worker or the skilled vacant firm is whether to invest resources in job training.[3] Such a decision is represented by an indicator function ξ_t^w, ξ_t^f respectively taking on values 0 or 1 for no training and training for worker and firm respectively.

Vacant firms and unemployed workers randomly match. At each date, u_t^i proportion of unemployed i-type workers meet v_t^j $(j = s, u)$ proportion of vacant firms. Let λ^{ij} be the probability that such a match consummates. The matching function is thus given by: [4]

$$M_t^{ij} = \lambda^{ij} u_t^i v_t^j \tag{2.1}$$

Based on the technology and the provisions of job training, three types of matching are conceivable: (i) a high skilled worker successfully matches a vacant high skilled firm, and produces output p^s, (ii) a low skilled worker successfully matches a vacant low skilled firm and produces p^u, (iii) a low skilled worker successfully matches a vacant high skilled firm and this vacant firm decides to train this worker by incurring a training cost. Each period a fixed fraction σ^i of existing matches in the skilled and unskilled sector die due to exogenous retirement or layoffs.

Let μ_t^{if} and μ_t^{iw} be the number of the i-th type firms and workers respectively.

$$\triangle u_t^s = \sigma^s(\mu_t^{sw} - u_t^s) - \lambda^{ss} u_t^s v_t^s + \xi_t^w u_t^u + \xi_t^f \lambda^{us} u_t^u v_t^s \tag{2.2}$$

$$\triangle u_t^u = [\sigma^u(\mu_t^{uw} - u_t^u) - \lambda^{uu} u_t^u v_t^u] - \xi_t^w u_t^u - \xi_t^f \lambda^{us} u_t^u v_t^s \tag{2.3}$$

$$\triangle v_t^s = \sigma^s(\mu_t^{sf} - v_t^s) - \lambda^{ss} u_t^s v_t^s \tag{2.4}$$

$$\triangle v_t^u = \sigma^u(\mu_t^{uf} - v_t^u) - \lambda^{u\,u} u_t^u v_t^u \tag{2.5}$$

$$\triangle \mu_t^{sw} = \xi_t^w u_t^u + \xi_t^f \lambda^{us} u_t^u v_t^s \tag{2.6}$$

$$\triangle \mu_t^{sf} = 0 \tag{2.7}$$

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[3]Since the focus of this paper is on job training, for simplicity we rule out the possibility of technology upgrade by unskilled firms to turn skilled.

[4]This matching function is known in the literature as a quadratic matching function following Diamond and Maskin (1979). Such a matching function can be motivated by the illustrative example borrowed from Mortensen and Pissarides (1998) that both matched and unmatched firms and households have a telephone book of all matched and unmatched agents on the other end of the market. A quadratic matching function may give rise to multiple equilibria. In our context, we break such multiplicity by invoking an initial distribution of skilled and unskilled workers and firms.

A few clarifications of the terms in the transition equations are in order. The transition equation (2.2) shows that the number of skilled unemployed increases when job separations occur (first term) and it decreases when an unemployed skilled worker successfully matches a vacant skilled firm (second term). Number of unemployed skilled workers also increases if unskilled workers complete self training (third term) . Finally vacant skilled firms successfully match with unskilled unemployed and invest in job training, and since during the training the firms and workers do not produce anything they are deemed as skilled unemployed (the fourth term).

Equation (2.3) follows by symmetry after observing (2.2). The transition equation (2.4) for the vacant skilled firms shows that the number of vacant firms increases when a job separation occurs (first term). The number of vacant firms decreases when a vacant skilled firm successfully matches with an unemployed skilled worker (second term)[5]. Similar reasoning applies to the transition equation (2.5) for vacant unskilled firms.

The transition equation for the skilled workers (2.6) means that more skilled workers evolve as more unskilled workers invest in job training (the first term), or more vacant skilled firms successfully match with unskilled unemployed and invest in job training (the second term). The transition equation (2.7) reflects that the number of high skilled firms is constant over time.

We focus on the steady state analysis only. There are four possible steady states for this system: (i) firms invest in training while workers do not, $\xi_t^f = 1, \xi_t^w = 0$,(ii) firms do not invest in training but workers do, $\xi_t^f = 0, \xi_t^w = 1$, (iii) both invest in training, $\xi_t^f = 1, \xi_t^w = 1$, (iv) none invest in training, $\xi_t^f = 0, \xi_t^w = 0$. Define the set of steady states in training as $\Pi = \{10, 01, 11, 00\}$ where the first element of each tuple is the training state of the firm, ξ_t^f and the second is the same of the worker, ξ_t^w. Let π stand for an element of the set Π.

In the following lemma, we prove that if at least one party invests in job training, all unskilled workers turn skilled and all unskilled jobs remain vacant.

Lemma 1 *The states where at least one of the agents invest in education (meaning $\pi \neq 00$) the stable steady state solutions of the transition equations are given by*

1. $u_\pi^u = 0, v_\pi^u = \mu_0^{uf}$,

2. $\mu_\pi^{sw} = 1, \mu_\pi^{uw} = 0,$

3. $\mu_\pi^{sf} = \mu_0^{sf} = 1 - \mu_\pi^{uf},$

4. $u_\pi^s = \overline{u}^s \equiv \frac{1}{2} \left(\left[\overline{\delta}^s - \eta^{ss} \right] + \sqrt{\left[\overline{\delta}^s - \eta^{ss} \right]^2 + 4\eta^{ss}} \right)$ and

5. $v_\pi^s = \overline{v}^s \equiv \overline{u}^s - \overline{\delta}^s$

where $\eta^{ss} = \frac{\sigma^s}{\lambda^{ss}}$ and $\overline{\delta}^s = 1 - \mu_0^{sf}.$

Proof of Lemma:. Appendix 1. ■

If at least one person invests in training, unskilled workers disappear in the economy. This means that the unemployment rate for unskilled workers goes to zero and all unskilled firms remain vacant. This explains the results (1), (2) and (3) of Lemma 1. On the other hand, there is still some natural rate of unemployment and vacancy in the skilled sector due to matching frictions which are characterized in results (4) and (5). Result (5) states that after training, the number of active skilled firms $\mu_0^{sf} - \overline{v}^s$ exactly balances the number of employed skilled workers $(1 - \overline{u}^s)$.

When nobody invests in training, the number of high skilled and low skilled workers does not change from its initial level. The following lemma formalizes it.

Lemma 2 : *The state where no-one invests (meaning $\pi = 00$) the stable steady state solutions of the transition equations are given by*

$$\mu^{iw} = \mu_0^{iw}; \mu^{iw} = \mu_0^{iw} \tag{2.8}$$

$$u_{00}^i = \frac{1}{2} \left(\left[\delta_0^i - \eta^{ii} \right] + \sqrt{\left[\delta_0^i - \eta^{ii} \right]^2 + 4\eta^{ii}\mu_0^{iw}} \right) \tag{2.9}$$

$$v_{00}^i = u_{00}^i - \delta_0^i \tag{2.10}$$

where $\delta_0^i = \mu_0^{iw} - \mu_0^{if}$ and $\eta^{ii} = \frac{\sigma^i}{\lambda^{ii}}$, $i = u, s.$

Proof of Lemma:. Appendix 1. ■

Lemmas (1) and (2), enable us to reduce the number of states to two, namely: (i) state of no training and (ii) state of training. Define

$$\begin{aligned} \overline{\xi} \equiv 1 - \left(1 - \xi^f \right) \left(1 - \xi^w \right) \quad &= 1 \quad \text{if at least one party invests in training} \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{2.11}$$

6

Then the vacancy and unemployment rates in each sector can be rewritten in a compact form as:

$$v_\pi^i = \left(1 - \overline{\xi}\right) v_{00}^i + \overline{\xi} \overline{v}^i \ , i = u, s \tag{2.12}$$

$$u_\pi^i = \left(1 - \overline{\xi}\right) u_{00}^i + \overline{\xi} \overline{u}^i, \ i = u, s \tag{2.13}$$

When at least one party invests in training, it also increases unemployment rate in the skilled sector because the number of skilled firms do not change while the number of skilled workers increase. Since there are more unemployed skilled workers, the probability of a skilled vacant firm matching with a skilled worker also increases. Consequently skill gap also decreases. This gives rise to an incentive for the skilled firms to invest in training. We have the following corollary summarizing these results.

Corollary 1 : a) The probability of matching with a skilled worker is higher in the state of training

$$\lambda^{ss} u_{00}^s < \lambda^{ss} \overline{u}^s$$

b) The skill gap decreases in the state of training

$$\mu_0^{if} - \mu_0^{iw} > \mu_0^{if} - 1$$

Proof of Corollary:. Appendix 1. ∎

The important implication of this corollary is that unlike Acemoglus and Pischke (1999) the matching probability is endogenous because it depends on the state of training. We will see later that this has important implications for the structure of wages depending on who undertakes training. In the next section, we turn to worker's and firm's decision problems which determine their training propensities.

3 Strategic Job Training

The job training decisions are made in a noncooperative, strategic environment. Let sc be the cost for training a worker. Let b be a common leisure value of any unemployed worker of any type, c be a common cost of keeping a production unit vacant and ω^i be the wage prevailing in the i-th sector. Unskilled workers while deciding to incur training costs take into account that even if they do not incur this cost, there is a chance of being hired by a skilled firm and getting trained subsequently.

A skilled firm while contemplating to train an unskilled worker internalizes the fact that the same worker may be exogenously separated from the firm after training. The job training thus appears as an equilibrium outcome of a dynamic game between workers and firms in a search environment.

There are eight types of agents in our economy: (i) unskilled employed, (ii) unskilled unemployed workers, (iii) skilled employed workers, (iv) skilled unemployed workers, (v) vacant skilled firms, (vi) vacant unskilled firms, (vii) active skilled, and (viii) active unskilled firms. Define the value functions of these eight types as: E^u, U^u, E^s, U^s, V^s, V^u, J^s, J^u respectively. Among these eight types, only unskilled unemployed workers (ii) and vacant skilled firms (v) can invest in job training and are thus deemed as active players in training decisions. The remaining six types of agents are passive in the sense that they do not involve in job training. It is assumed that all engaged skilled and employed unskilled workers use a fixed amount of time for production. This time is indivisible in the sense that it cannot be divided between production and investment in training. This explains why all engaged skilled firms and employed unskilled workers cannot invest in training. Since we rule out the possibility of a technological upgrade by an engaged unskilled firm, it is not worthwhile for such a firm to invest in job training. Nevertheless, these passive agents experience externality from the training decisions of the active agents because skilled wages are influenced by training. The steady state value functions of these passive agents thus depend on the Nash equilibrium arising from the strategic training decisions of unskilled unemployed worker and vacant skilled firms. To see this clearly, let us first spell out U^u and V^s.

3.1 Value Functions of the Active Players

The only initial state of interest here is the state of no training because otherwise the mass of unskilled workers goes to zero by virtue of Lemma 1. An unskilled unemployed worker collects unemployment benefit at present and faces two choices: to train himself or not to train himself. If he goes for self-training, there are two prospects: (i) the prospect of being matched with a vacant skilled firm with probability $\lambda^{ss} v_\pi^s$ or (ii) the prospect of no such match. If he does not go for self-training, three possibilities lend themselves: (a) he can be matched with either a vacant unskilled firm with probability $\lambda^{uu} v_\pi^u$, (b) he could remain unemployed with a prospect of being matched with a vacant skilled firm who may impart training to him with a probability $\lambda^{us} v_\pi^s$. We assume that the labour supply is indivisible in the sense that while training, the worker must spend full time in it and cannot produce any output. And (c), he may simply remain unemployed without any match whatsoever. If

8

he indeed matches with a vacant skilled firm who imparts training to him, during the training period he does not produce anything or does not receive any wages. During this training state, his status is deemed to be skilled unemployed.

The value function of an unskilled worker is thus given by:

$$U^u\left(\xi^f, \xi^{w*}\right) = b - \xi^{w*}sc + \xi^{w*}\beta\left[\lambda^{ss}v^s_\pi E^s + (1 - \lambda^{ss}v^s_\pi)U^s\right] +$$

$$(1 - \xi^{w*})\beta\begin{bmatrix} \lambda^{uu}v^u_\pi E^u + \xi^{f*}\lambda^{us}v^s_\pi U^s \\ +(1 - \lambda^{uu}v^u_\pi - \xi^{f*}\lambda^{us}v^s_\pi)U^u\left(\xi^f, \xi^{w*}\right) \end{bmatrix} \qquad (3.14)$$

where $\xi^{w*} = \arg\max_{\xi^w} U^u\left(\xi^f, \xi^w\right)$.
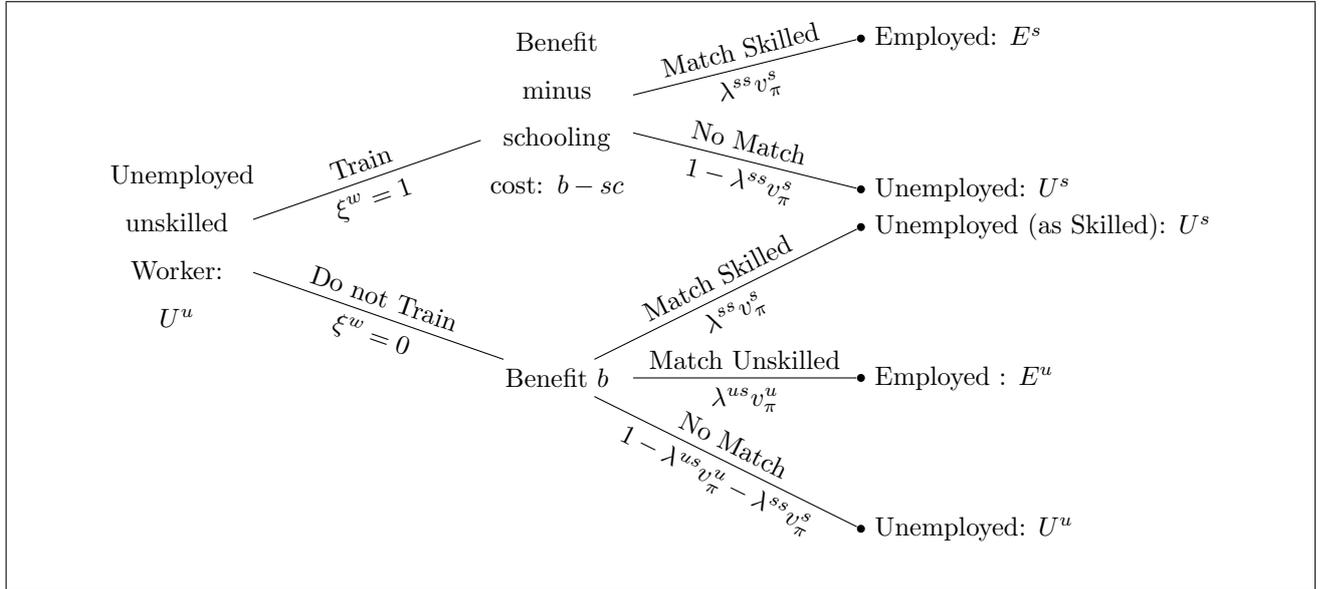
The flow chart below makes it clear

Figure 1: Flow chart of Unskilled Unemployed Worker.

Next consider the formulation of V^s. A vacant skilled firm currently incurs the sunk cost c of keeping its unit vacant and also a training cost of sc depending on whether it decides to train an unskilled worker. If it decides to train, the unskilled worker turns skilled with a probability $\lambda^{us}u^s_\pi$. As described earlier no production takes place during training due to indivisibility of labour. Therefore the firm is deemed vacant during this period. The firm can still have a chance to match with a born skilled worker with probability $\lambda^{ss}u^s_\pi$. If the matching does not work out, the vacant skilled firm remains inactive with complement probability $(1 - \lambda^{ss}u^s_\pi - \lambda^{us}u^u_\pi)$. On the other hand, if the skilled vacant firm does not spend on training, there is still a chance of a match with a skilled worker with

probability $\lambda^{ss}u_\pi^s$ and remains vacant with probability $(1 - \lambda^{ss}u_\pi^s)$. The value function of the vacant skilled firm is thus given by:

$$V^s\left(\xi^{f*}, \xi^w\right) = -c - \xi^{f*}sc + \xi^{f*}\beta \left[\begin{array}{c} \lambda^{ss}u_\pi^s J^s + \lambda^{us}u_\pi^u V^s\left(\xi^{f*}, \xi^w\right) \\ + \left(1 - \lambda^{ss}u_\pi^s - \lambda^{us}u_\pi^u\right) V^s\left(\xi^{f*}, \xi^w\right) \end{array} \right] \tag{3.15}$$
$$+ \left(1 - \xi^{f*}\right)\beta \left[\lambda^{ss}u_\pi^s J^s + \left(1 - \lambda^{ss}u_\pi^s\right) V^s\left(\xi^{f*}, \xi^w\right)\right]$$

where $\xi^{f*} = \arg\max_{\xi^f} V^s\left(\xi^f, \xi^w\right)$.

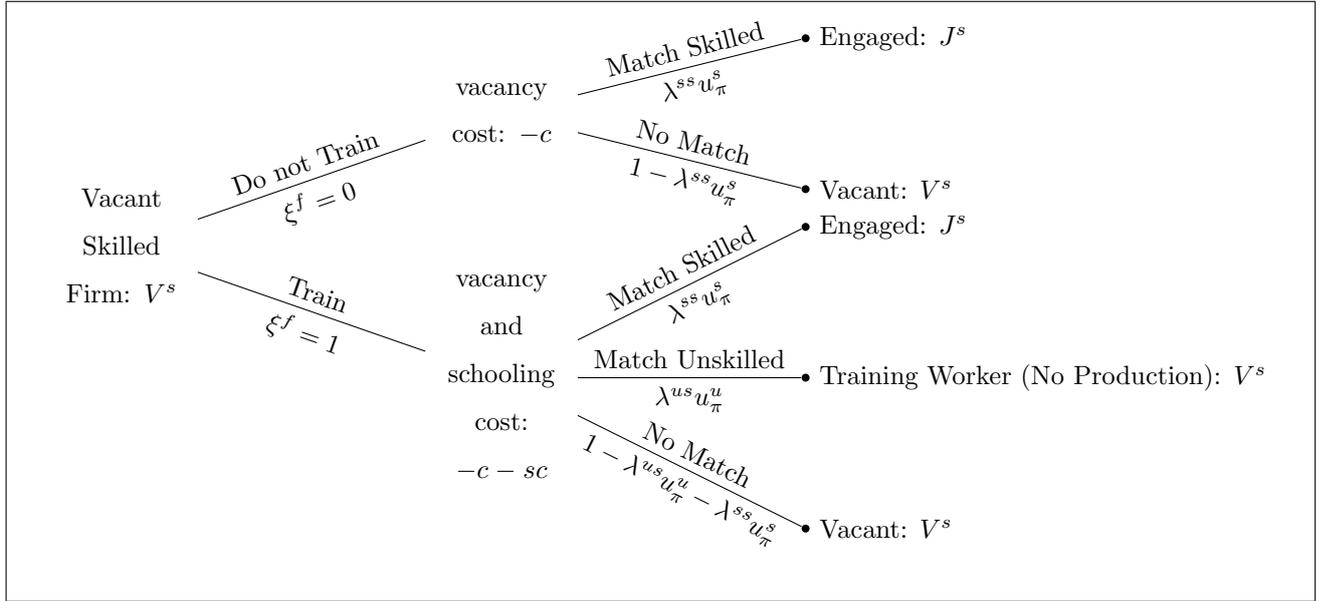**Figure 2:** Flow chart of Vacant Skilled Firm.

It is important to observe that the skilled vacancy rate v_π^s and the unemployment rate u_π^s depend on the state of training via (2.12) and (2.13).

3.2 Value Functions of the Passive Agents

We next formulate the value functions of the remaining six types of agents who are deemed as passive since they do not undertake any training decisions. However, each of their values depends on the training decisions ξ^f and ξ^w of skilled vacant firms and unskilled unemployed workers through the vacancy and unemployment rate v_π^i and u_π^i (in (2.12) and (2.13)) and wages which will be specified in the next section. As all these passive agents are not involved in the training decision, we do not hereafter write the values of these agents as functions of ξ^f and ξ^w.

10

A skilled employed worker can earn a wage ω^s today and faces two scenarios: (i) stay employed in the next period with a probability $(1 - \sigma^s)$, or (ii) join the pool of skilled unemployed with probability σ^s. A skilled unemployed worker collects unemployment benefits, b today and faces the prospect of being matched with a vacant skilled firm with probability $\lambda^{ss} v_{00}^s$ when there is no investment in training and with probability $\lambda^{ss} \overline{v}^s$ when there is investment in training.[6] Thus, the value functions of skilled employed and unemployed are given by:

$$E^s = \omega^s + \beta \left[\sigma^s U^s + (1 - \sigma^s) E^s \right] \tag{3.16}$$

$$U^s = b + \beta \left[\lambda^{ss} v_\pi^s E^s + (1 - \lambda^{ss} v_\pi^s) U^s \right] \tag{3.17}$$

Likewise an unskilled employed worker has the following value function:

$$E^u = \omega^u + \beta \left[\sigma^u U^u + (1 - \sigma^u) E^u \right] \tag{3.18}$$

An active unskilled firm produces p^u and after paying wage ω^u to the worker faces two prospects next period: (i) stay active with a probability $(1 - \sigma^u)$ or (ii) join the pool of vacant unskilled firms with a probability σ^u. An inactive (vacant) unskilled firm incurs the vacancy cost c and faces the prospect of being matched with an unemployed unskilled worker with a probability $\lambda^{uu} u_\pi^u$ or stay vacant with a probability $(1 - \lambda^{uu} u_\pi^u)$. The value functions for the unskilled firms are thus given by:

$$J^u = p^u - \omega^u + \beta \left[\sigma^u V^u + (1 - \sigma^u) J^u \right] \tag{3.19}$$

$$V^u = -c + \beta \left[\lambda^{uu} u_\pi^u J^u + (1 - \lambda^{uu} u_\pi^u) V^u \right] \tag{3.20}$$

Likewise an active skilled firm has a value function:

$$J^s = p^s - \omega^s + \beta \left[(1 - \sigma^s) J^s + \sigma^s V^s \right] \tag{3.21}$$

Wage Determination

The wage in each sector is determined by a Nash bargaining:

$$\omega^s \left(\xi^{f*}, \xi^{w*} \right) = \arg \max_{\omega^s} \left(E^s - U^s \right)^{1-\theta} \left(J^s - V^s \left(\xi^{f*}, \xi^{w*} \right) \right)^\theta \text{ and} \tag{3.22}$$

$$\omega^u \left(\xi^{f*}, \xi^{w*} \right) = \arg \max_{\omega^u} \left(E^u - U^u \left(\xi^{f*}, \xi^{w*} \right) \right)^{1-\theta} \left(J^u - V^u \right)^\theta \tag{3.23}$$

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[6]It is assumed that the skilled unemployed worker does not search for a job in the unskilled sector.

respectively where θ is a non-negative fraction representing the bargaining strength of the firm and $\left(\xi^{f*}, \xi^{w*}\right)$ are equilibrium strategies. The Nash bargaining wage is basically the weighted average of the flow excess return of the firm from employing a worker vis-a-vis keeping the position vacant and the flow excess return of the worker taking a job vis-a-vis staying unemployed.

Characterization of Equilibrium

We now define formally a Nash equilibrium for our economy which reflects the interdependence of workers and firms through the training decisions of the active players.

Definition: Given the initial state of no training and a training cost of sc, a Nash equilibrium in training satisfies the following conditions:

(i) An unskilled unemployed worker chooses the training decision ξ^w optimally taking the training decision (ξ^f) of the vacant skilled firm as given. In other words, $U^u\left(\xi^{f*}, \xi^{w*}\right) \geq U^u\left(\xi^{f*}, \xi^w\right)$

(ii) A vacant skilled firm chooses the training decision ξ^f optimally taking the training decision (ξ^w) of the unskilled worker as given. In other words, $V^s\left(\xi^{f*}, \xi^{w*}\right) \geq V^s\left(\xi^f, \xi^{w*}\right)$.

(iii) Given the training decisions of unskilled workers and vacant skilled firms, other workers behave optimally and solve (3.16), (3.17), (3.18).

(iv) Given the training decisions of unskilled workers and vacant skilled firms, other firms behave optimally and solve (3.19), (3.20) and (3.21).

(v) Given the optimal training decisions of workers and firms, wages are determined by Nash bargaining as in (3.22) and (3.23).

Equilibrium Wage Structure

A key result now follows that shows that the skilled wage structure depends on who invests in training.

Lemma 3 *Skilled wage is higher in the state when firms undertake training compared to the state where workers undertake training,*

$$\omega^s\left(1,0\right) > \omega^s\left(0,1\right). \tag{3.24}$$

Proof of Lemma:. Appendix 2. ∎

A vacant skilled firm in our model becomes engaged by bearing the sunk cost of training an unskilled worker. Since it has already borne this cost, it is incentive compatible for the firm to keep the worker by paying a higher wage to keep this worker. On the other hand, when an unskilled

unemployed worker bears the cost of training, he just acquires the same level of skill as a born skilled worker. This means that he cannot strike a higher wage bargain and shift the cost of training to the employer. This explains why skilled wage is higher when firms invest in training.

3.3 Coordination Failure in Training: No Training Equilibrium

Depending on the schooling cost, various equilibria are possible where either the firm or worker may or may not choose to invest in training. We are particularly interested in a Nash equilibrium where none may invest in training while it is socially optimal that at least someone invests in training. Such a scenario is deemed to be a coordination failure among agents because private agents do not internalize certain social benefits of training. To understand the nature of this coordination failure, we proceed in two steps. First, we characterize the range of schooling costs for which there is a Nash equilibrium in no training. Second, we set up a social planning problem where the fictitious social planner internalizes all the benefits and costs of training and shows the conditions under which it is socially optimal to invest in training but not privately optimal.

Given the initial state of no training and a training cost of sc, a Nash equilibrium where nobody invests in training satisfies the following two conditions:

1. Given that firms do not bear training cost, workers will not pay if

$$U^u(0,1) < U^u(0,0). \qquad (3.25)$$

2. Given that workers do not pay, a vacant skilled firm will not incur the training cost if

$$V^s(1,0) < V^s(0,0). \qquad (3.26)$$

Since $U^u(0,1)$ and $V^s(1,0)$ are monotonically decreasing in sc, there exists a threshold training cost sc_w^{n*} for which (3.25) holds as equality.[7] The worker does not pay for training when the firm does not pay if

$$sc > sc_w^{n*} \qquad (3.27)$$

Likewise, there exists a threshold schooling cost sc_f^{n*} for which (3.26) holds as equality. Given that the worker does not pay, the firm does not pay for training if

$$sc > sc_f^{n*} \qquad (3.28)$$

---

[7]From (A20) it is obvious $\frac{\partial U^u(0,1)}{\partial sc} = -1$ and from (A19) $\frac{\partial V^s(1,0)}{\partial sc} = -\frac{1}{1-\beta}\left(1 - \frac{\theta\beta\lambda^{ss}\overline{u}^s}{1-\beta(1-\sigma^u-\lambda^{ss}\overline{u}^s)}\right) < 0$, which establishes the monotonicity of $U^u(0,1)$ and $V^s(1,0)$.

Appendix 2 provides an algebraic derivation of these two thresholds.

Based on the above analysis we have the following lemma.

Proposition 2 *If the training cost per pupil is such that*

$$sc > \max(sc_w^{n*}, sc_f^{n*}) = sc^{n*}(say) \tag{3.29}$$

neither vacant skilled firm nor the unemployed unskilled worker finds it worthwhile investing in training.

The training propensity of firms and workers depends on the relative bargaining strength parameterized by θ. We have the following key proposition.

Proposition 3 *Properties of threshold costs*

i) $sc_w^{*n} = 0$ *when* $\theta = 1$

ii) $sc_f^{*n} \geq 0$ *and* $sc_f^{*n} = 0$ *when* $\theta = 0$

iii) $\frac{\partial sc_w^{*n}(\theta)}{\partial \theta} < 0$

iv) $\frac{\partial sc_f^{*n}(\theta)}{\partial \theta} > 0$

In other words, worker's propensity to self train is higher when they have greater bargaining strength. Firms are willing to bear the training cost more if they have greater bargaining strength. These results are similar in spirit to Acemoglu and Pischke (1999) because the bargaining parameter θ proxies the degree of wage compression in Acemoglu and Pischke (1999).

4 Socially Optimal Training

We next turn to a social planning problem where the social planner internalizes the costs and benefits of training. The planner mandates whether a firm or a worker should spend on job training while internalizing the benefits and costs of keeping workers unemployed and positions vacant. We focus on steady states only. The social planner takes the steady state configurations of the relevant state variables, $u_t^s, u_t^u, v_t^s, v_t^u, \mu_t^{sw}, \mu_t^{sf}$ as given. Recall from Lemma 1 and 2 that the steady states of the economy are entirely dependent on the state of training, π[8].

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[8]In principle, the entire history of training should comprise the current state facing the planner. However, given the absorbing nature of the state (meaning when either the worker or the firm invests in training, an unskilled worker or firm turns permanently skilled next period), the current state is thus summarized only by the current state of job training.

Since the steady state values of these state variables are also functions of λ^{ij} and σ^i, the social planner internalizes the search frictions while reaching a decision about job training. There are four possible states of training for $\pi \in \Pi$. The only relevant initial steady state is when there is no past investment in training (meaning $\pi = 00$) because we know from Lemma 1 that otherwise everybody is trained to start with, thus making job training redundant. Starting from this initial state, the planner can mandate four possible actions: (i) no change meaning $\xi^f = 0$, and $\xi^w = 0$, (ii) ask only firms to invest in training, $\xi^f = 1$, and $\xi^w = 0$, (iii) ask only workers to invest in training, $\xi^f = 0$, and $\xi^w = 1$ and (iv) ask both to invest in training, $\xi^f = 1$, and $\xi^w = 1$. The planner chooses the action that gives the best societal value.

Define r_π as the steady state proceeds to the social planner at the state π. This can be written as:

$$r_\pi = \sum_{i=s,u} \left[p^i(\mu_\pi^{iw} - u_\pi^i) + bu_\pi^i - cv_\pi^i \right]. \tag{4.30}$$

In other words, the steady state proceeds to the planner is the sum total of outputs from skilled and unskilled sectors plus the total leisure benefits to the skilled and unskilled workers minus the total vacancy costs of skilled and unskilled units. Note that the social planner internalizes the utility value of leisure and vacancy costs in the proceeds, r_π. In contrast, in a decentralized Nash economy, a firm does not internalize the leisure value of workers and neither does the worker internalize the vacancy cost of firms.

We have the following lemma.

Lemma 4 *Define* $\overline{r} \equiv p^s(1 - \overline{u}^s) + b\overline{u}^s - c(\overline{v}^s + \overline{v}^u)$, *then* $r_{01} = r_{10} = r_{11} = \overline{r}$.

Proof of Lemma:. Replacing the values of $\mu_\pi^{iw}, u_\pi^i, u_\pi^i, v_\pi^i$, in (4.30) for $i = u, s$ and $\pi \neq 00$, from Lemma 1 gives the result. ∎

In other words, the social planner is indifferent who invests in training. If at least one party invests in training, the steady state social proceeds from this is given by $\overline{r}$ which is the skilled sector output plus benefits of skilled unemployed people minus the vacancy costs.

This considerably simplifies the social planner's problem. Starting from a no training state, if the social planner decides not to mandate any training programme, the social value is given by $\frac{r_{00}}{1-\beta}$ where

$$r_{00} = \sum_{i=u,s} \left[p^i(\mu_0^{iw} - u_{00}^i) + (b - c)u_{00}^i \right]$$

15

If the planner mandates a training programme, the society incurs a training cost, $sc.u_{00}$ today but in the next period it lands into a state where everybody is skilled and the social value is $\frac{\bar{r}}{1-\beta}$

The planner thus initiates a change from no training to positive training if

$$\frac{r_{00}}{1-\beta} \leq r_{00} - sc.u_{00}^u + \beta\frac{\bar{r}}{1-\beta}$$

which means that

$$sc.u_{00}^u \leq \frac{\beta}{1-\beta}[\bar{r} - r_{00}] \tag{4.31}$$

The planner finds it worthwhile mandating job training if the training cost (left hand side of (4.31)) exceeds the annuity value of the proceeds differential when the planner initiates a change from no training to positive training (the right hand side of (4.31)).

Based on (4.31) it immediately follows that the social planner mandates investment in training if the schooling cost (sc) is below a certain threshold given by:

$$sc < \frac{\beta}{1-\beta}\frac{[\bar{r} - r_{00}]}{u_{00}^u} = sc^{p*}(say). \tag{4.32}$$

Source of Coordination Failure

It follows from (3.29) and (4.32) that for a range of schooling costs, $sc^{n*} < sc < sc^{p*}$, it is socially beneficial to institute a job training programme but it is not privately incentive compatible. While undertaking a social cost benefit analysis of training, the planner internalizes the gain in skilled sector output and the loss of unskilled sector output, the saving of vacancy cost in each sector, the loss of the worker's leisure time as well as the search frictions. In a market economy, firms and workers do not internalize all these benefits and costs in the same way the social planner does. For example, an unskilled worker does not fully internalize the saving of a skilled firm's cost of keeping positions vacant while deciding about joining a skill centre. Likewise, a vacant firm will not pay attention to a worker's loss of leisure time if they train unskilled workers. This conflict of interest is at the very root of the coordination failure in training. There could be underinvestment in training which is not socially optimal.[9]

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[9]There is a theoretical possibility of overinvestment in training when it is socially undesirable to institute a job training programme but the private sector incurs this cost anyway. This happens when $sc^{p*} < sc < sc^{n*}$. In our calibrated model that we report next, this case does not arise because $sc^{p*} > sc^{n*}$ for empirically plausible parameter values.

4.1 Redistributive Effects of Training

The social planning problem provides an important lesson that someone should pay for training for a range of schooling cost. Who should pay for training? The answer is not obvious because who bears the cost of training has redistributive effects on the values of skilled firms and workers. The next proposition makes it clear.

Proposition 4 $J^s(1,0) < J^s(0,1),\ V^s(1,0) < V^s(0,1),\ \ E^s(1,0) > E^s(0,1),\ U^s(1,0) > U^s(0,1).$

Proof. Appendix 2. ∎

When firms pay for training, skilled wages are higher than a scenario when workers pay for training. For reasons mentioned in Lemma 3, the steady state values of both vacant and engaged skilled firms are thus lower. All skilled workers, on the other hand, experience a higher expected wages and thus enjoy a gain in their steady state values. All vacant unskilled firms experience no change in their values because all unskilled workers turn skilled regardless of who pays for training. The overall effect is that skilled firms lose (gain) and skilled workers gain (lose) when firms (workers) invest in training.

The aggregate welfare effects of training should take into account this redistributive effect of welfare. In the next section we turn to a calibration of our model to quantify the welfare effect of instituting training in the economy.

5 Quantitative Analysis

In this section, we report a quantitative estimate of the welfare loss due to private underinvestment in training. There are three steps of this exercise. First, based on the available studies and model steady state properties, we compute the baseline estimates for our structural parameters of the model. Second, based on these baseline parameter values, we compute the schooling cost thresholds for the social planner and the private sector, namely sc^{p*}, sc_f^{n*}, sc_w^{n*}. Finally, using these calibrated cost thresholds, we compute the welfare effects of training.

5.1 Fixing parameter values

There are 12 structural parameters, namely $\left\{\mu_0^{ws}, \mu_0^{fs}, p^s, p^u, b, c, \beta, \lambda^s, \lambda^u, \sigma^s, \sigma^u, \theta\right\}$ characterizing the economy. Because of the stylized nature of the model, it is difficult to find estimates from microeconomic studies for all these 12 parameters for the UK economy within a common timeframe.

Since we are calibrating the steady state properties of the model, we take the liberty of choosing available estimates for different time periods under the assumption that the UK economy is in a steady state equilibrium. We also assume that the steady state properties of the UK economy are similar to advanced OECD countries. This justifies the selection of OECD estimates for a few model parameters in the absence of any suitable UK estimate.

The estimate of β is 0.99 as in Shimmer (2005). We assume that the job separation rates in the skilled and unskilled sector are the same and set it at 0.1 as in Shimmer.

As there are four steady states in the model, (0,0), (0,1), (1,0), (1,1), the issue arises which steady state should be used for baseline calibration. Since the goal of the paper is to understand the reasons for the failure of job training, the relevant baseline steady state is chosen to be (0,0) which is the state of no training.

According to Nickell and Bell (1994), for UK the proportion of labour force in the skilled sector is 36.8% and in the unskilled sector, it is 28.2% . The remaining labour force is called non-employment which includes discouraged unskilled workers. Since in our model we only have skilled and unskilled workers in the labour force, we normalize Nickell and Bell's estimates to arrive at the proportion of skilled and unskilled workers in the labour force. This means that μ_0^{ws} =36.8%/(36.8%+28.2%) and $\mu_0^{wu} = 1 - \mu_0^{ws}$.

The unemployment benefit parameter b is proxied by the value of leisure time taken from Shimer (2005) and is fixed at 0.4. Using the same study, we fix the cost of vacancy, c and the bargaining parameter, θ at 0.21 and 0.72 respectively. Acemoglu and Zilibotti (2000) compute the value added per worker in low, medium and high skilled sectors in rich and poor countries. The ratio of the value added per worker in high to low skilled sectors is about 2.12 for both rich and poor countries. Based on their study p^s/p^u, is thus fixed at 2.12.

The remaining parameters $\mu_0^{sf}, \mu_0^{uf}, \lambda^{ss}, \lambda^{uu}, \lambda^{us}$ are calculated using the steady state properties of the model. Without any loss of generality, for the purpose of calibration we assume that $\lambda^{us} = \lambda^{uu}$, which means that the probability of an unskilled worker meeting a skilled vacant firm is the same as the probability of an unskilled worker meeting an unskilled firm.

We need four steady equations to solve for $\mu_0^{sf}, \mu_0^{uf}, \lambda^{ss}, \lambda^{uu}$. The labour market clearing conditions in the skilled and unskilled sectors (Lemma 2) imply that,

$$\mu_0^{iw} - u_{00}^i = \mu_0^{if} - v_{00}^i, i = u, s \tag{5.33}$$

From Lemma (2) we get:

$$\lambda^{ii} = \sigma^i \frac{\mu^{iw} - u^i_{00}}{\left[u^i_{00}\right]^2 - \delta^i u^i_{00}}, i = u, s \tag{5.34}$$

Given the estimates of the unemployment and vacancy rates $u^s_{00}, u^u_{00}, v^s_{00}, v^u_{00}$, our four equations in (5.33) and (5.34) can be solved for $\mu^{sf}_0, \mu^{uf}_0, \lambda^{ss}, \lambda^{uu}$. Estimates of u^s_{00}, u^u_{00} came from Nickell and Bell (1994). They estimate these unemployment rates for two levels of education, low education and high education as discussed earlier for the two years 1973 and 1991. We use their estimates for the relatively recent year 1991 and normalize these by the proportion of workers in the labour force in each group to arrive at our estimates of u^s_{00} and u^u_{00}. Doing so, we obtain, $u^s_{00} = 8.92\%$ and $u^u_{00} = 24.15\%$.

Regarding the calibration of vacancy rates v^s_{00}, v^u_{00}, observe from the market clearing condition (5.33) that

$$u^s_{00} + u^u_{00} = v^s_{00} + v^u_{00} \tag{5.35}$$

Our calibrated vacancy rates must respect this vacancy-unemployment identity (5.35). To this end, we use the Office of National Statistics (ONS) database to get some prior estimates of v^s_{00}, v^u_{00}. We call these estimates $v^{s,ONS}_{00}$ and $v^{u,ONS}_{00}$. ONS provides the vacancies per 100 jobs for 19 occupations. We select 9 of these occupations as skilled. The remaining occupations are classified as unskilled.[10]

Evidently this classification of occupations among skilled and unskilled is somewhat arbitrary. Our motivation for this classification is to remain consistent with the classification of Nicole and Bell among low and high education workers. We work on the assumption that 9 jobs selected as skilled require some form of formal education.

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[10]Skilled occupations are namely, (i) Real Estate Activities, (ii) Professional, Scientific & Technical, (iii) Admin & Support Service Activities, (iv) Public Administration & Defence, (v) Education, (vi) Human Health & Social Work, (vii) Arts, Entertainment & Recreation, (viii) Information and communication, (ix) Finance and Insurance. Unkilled occupations are: (i) mining and quarrying, (ii) manufacturing, (iii) Electricity and gas supply, (iv) water suppply and sewage, (v) construction, (vi) wholesale and retail, (viii) transportation and storage, (ix) accommodation, (x) other services (xi) total services. The last two categories are unspecified occupations. Including these last two in the skilled category does not change the result significantly.

Using these ONS estimates, we normalize v_{00}^s, v_{00}^u as follows:

$$v_{00}^s = \frac{v_{00}^{s,ONS}}{v_{00}^{s,ONS} + v_{00}^{u,ONS}}(u_{00}^s + u_{00}^u)$$

$$v_{00}^u = \frac{v_{00}^{u,ONS}}{v_{00}^{s,ONS} + v_{00}^{u,ONS}}(u_{00}^s + u_{00}^u)$$

These normalized estimates for vacancy rates are model consistent and satisfy the identity (5.35).

Table 1 summarizes the baseline parameter values.

[Insert Table 1]

5.2 Welfare Effects of Training and Policy Implications

How does investment or lack of investment in training impact the welfare of workers and firms? Denote the welfare of all workers in the state π as W_π^w and likewise denote the welfare of all firms as W_π^f. We thus have:

$$W_\pi^w = \sum_{i=s,u} (\mu_\pi^{iw} - u_\pi^i)E^i + u_\pi^i U^i \tag{5.36}$$

$$W_\pi^f = \sum_{i=s,u} \left[(\mu_\pi^{if} - v_\pi^i)J^i + v_\pi^i V^i\right] \tag{5.37}$$

The welfare effect of training depends on who invests in training. We calculate the values of firms and workers by setting the schooling cost exactly equal to the corresponding training cost threshold. For example, when firms invest in training we set $sc = sc_f^{n*}$, at which a vacant skilled firm is just indifferent between training or no training. Likewise, when workers invest in training, we set $sc = sc_w^{n*}$.

We next turn to the details of the calculations of the welfare effects of training, The training decision of either firm or worker has a spillover effect on the values of all other firms and workers in the economy through strategic interdependence that impacts the aggregate welfare of all workers and firms as spelled out in (5.36) and (5.37).

Table 2 summarizes the changes in the welfare of firms and workers for the baseline model. When an unskilled worker alone invests in training, both firms and workers gain in welfare by 11% and 3%

respectively. The welfare of the whole economy (denoted as $W_\pi^e = W_\pi^w + W_\pi^f$) consisting of all workers and firms increases by 7%. On the other hand, when a vacant skilled firm alone invests in training, firms lose by about 21 % and workers gain by 23% while the overall welfare increases by 3%.[11]

[Insert Table 2]

The estimate of the welfare gain in a command economy reported in the last column of Table 2 is 8% [12]. Since the social planner internalizes all the private costs and benefits, not surprisingly the welfare gain from the institution of a job programme is higher in a planned economy than in a decentralized economy. Comparison of the last two columns of Table 2 reveals that when workers undertake a self financed training programme, the welfare gain in a decentralized economy comes close to the gain that one observes in a planned economy. On the other hand, the discrepancy between private and social welfare gains is much higher when firms impart training to workers. This discrepancy between private and social welfare gain is a measure of efficiency loss in a decentralized economy due to the coordination failure.

Tables 3 and 4 report the sensitivity analysis of these estimates of welfare gain when two crucial policy parameters, namely p^s/p^u and b change. A change in p^s/p^u raises the welfare gain in a private economy when either the firm or worker adopts training. The gain is higher when the firm initiates training. Change in unemployment benefit has little effects on the welfare gain from training regardless of who spends on training.

[Insert Tables 3 and 4]

The upshot of this welfare analysis is that the welfare gain is at its maximum if workers are given incentive to initiate training. The policy implication is that providing training subsidy directly to workers can induce workers to undertake training. Providing subsidy to skilled firms (that raises p^s/p^u) can also help as it results in an overall welfare gain.

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[11] The overall change in welfare is less than 10% because aggregate welfare is the weighted sum of workers' and firms' welfare. These weights change with respect to the state of training. These weight are also quite sensitive to training because due to training all unskilled workers turn skilled.

[12] The proportional change in the planner's welfare due to the institution of training is given by:

$$\frac{W_\pi^s - W_{00}^s}{W_{00}^s} = \frac{\beta \left(\overline{r} - r_{00} \right)}{r_{00}} = (1 - \beta) \frac{sc^{p*} u_{00}^u}{r_{00}}$$

5.3 Skill Premium and Income Inequality

In our model, the skill premium, (defined as the ratio skilled to unskilled wages, w_{00}^s/w_{00}^u) as shown in Tables 5 and 6, is about 50% in a state of no voluntary training programme for the baseline parameter values. This is of the same order of magnitude reported by Felstead et al. (2002) and Nickel and Bell (1994) A higher skilled:unskilled productivity gap (higher p^s/p^u) raises the skill premium because the skilled wage increases more than unskilled wages to reflect this productivity difference. On the other hand, a more liberal unemployment benefit (b) lowers the skill premium as evident from (A7) [13]. Since in our stylized model everyone becomes skilled following a job training, trivially the skill premium disappears after training.

How does a job training programme impact the income inequality? Tables 7 and 8 compare the Gini coefficient for economies with no training and positive training. Before the job training, the inequality is measured across four groups of workers, skilled and unskilled employed earning wages w_{00}^s and w_{00}^u respectively, skilled and unskilled unemployed collecting unemployment benefits b where $b < w_{00}^u < w_{00}^s$. A job training programme eliminates unskilled workers. The Gini coefficient in a state of no training is about 0.38, which is similar to the gini coefficient reported by National Statistics (between 0.36 to 0.34)[14] . A job training programme increases this income inequality by 5 to 15%, depending on who pays for training. Increase in inequality is more if firms invest in training rather than workers. This happens because the wage increase is more when firms invest in training for the reasons described earlier. Changes in productivity ratio and benefits have minor effects on the Lorenz ratio. Figure 1 plots the Lorenz curves for the two states of training setting the parameters at the baseline levels.

<div align="center">

[Insert Table 5]

[Insert Table 6]

[Insert Table 7]

[Insert Table 8]

[Insert Figure 3]

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[13]It is easy to verify that $\frac{\partial(w_{00}^s/w_{00}^u)}{\partial b} < 0$ if $p^s > p^u$.

[14]See http://www.statistics.gov.uk/cci/nugget.asp?id=332

6 Conclusion

In this paper, we attempt to explain two apparently conflicting stylized facts in the UK labour market. First, there is an acute skill shortage in the UK economy. High skilled positions remain vacant for a long time while there is an excess supply of intermediate skills. Second, there exists a substantial high to low skill premium. There is an unexploited profit opportunity in the high skilled sector while neither the worker nor the firm appears to take advantage of these through job training. We propose an explanation of this anomaly in terms of a coordination failure of firm's and worker's decisions regarding job training. Our model demonstrates that while it is socially optimal to invest in job training, the private sector may not internalize this benefit. As a result, there could be voluntary underinvestment in training. The degree of underinvestment depends crucially on the bargaining strength of the workers vis-a-vis firms.

Our model and the quantitative analysis predicts that a job training programme can increase social welfare and reduce skill gap. The welfare gain is higher if, instead of firms, workers bear the cost of training. In a state of no training, a skill premium of the order of 50% can arise which can be eliminated by instituting a training programme. A voluntary underinvestment in training thus reflects a state of social inefficiency which a benevolent government can correct by a public policy package such as output subsidy to skilled firms and less liberal unemployment benefits. A job training programme without an accompanying skill upgrade of firms could increase skilled unemployment and exacerbate the income inequality.

Appendix 1:

Proof of Lemma 1: Solve μ^{uw} and μ^{uf} . Using (2.6) and (2.7) we get: If $\left(\xi_t^f, \xi_t^w\right) \neq (0,0)$, conjecture a solution $\mu^{uw} = 0$. Since $0 \leq u^u \leq \mu^{uw}$ this means $u^u = 0$ as well [15]. Also $\mu^{sf} = \mu_0^{sf} = 1 - \mu^{uf}$. The steady state solutions using (2.5) and (2.3) using the previous solutions we have:

$$\sigma^u(\mu^{uw} - u^u) = \lambda^{uu}u^uv^u + \lambda^{us}u^uv^s \tag{A1}$$

$$\sigma^u(\mu^{uf} - v^u) = \lambda^{u\,u}u^uv^u \tag{A2}$$

implying $v^u = \mu_0^{uf}$.The other solutions satisfies the (A1) and (A2) as well. Plugging in $u^u = 0$ in equation (2.2) and (2.4) we solve for u^s and v^s from as:

$$\sigma^s(\mu^{sw} - u^s) = \lambda^{ss}u^sv^s \tag{A3}$$

$$\sigma^s(\mu^{sf} - v^s) = \lambda^{ss}u^sv^s \tag{A4}$$

$\mu^{sw} - u^s = \mu^{sf} - v^s$ or $v^s = u^s - \left(\mu^{sw} - \mu^{sf}\right) = u^s - \delta^s$, where $\delta^s = \left(\mu^{sw} - \mu^{sf}\right)$ is the equilibrium mismatch between skilled workers and skilled firms. Use (A3) and (A4) to get:

$$\sigma^s(\mu^{sw} - u^s) - \lambda^{ss}u^s\left[u^s - \delta^s\right] = 0$$

$$u^s\left[u^s - \delta^s\right] - \eta^{ss}(\mu^{sw} - u^s) = 0$$

$$[u^s]^2 - (\delta^s - \eta^{ss})u^s - \eta^{ss}\mu^{sw} = 0$$

where $\eta^{ss} = \frac{\sigma^s}{\lambda^{ss}}$. Then solving the quadratic system we have

$$u^s = \frac{1}{2}\left([\delta^s - \eta^{ss}] + \sqrt{[\delta_0^s - \eta^s]^2 + 4\eta^{ss}\mu^{sw}}\right)$$

$$v^s = u^s - \delta^s$$

Plugging in the values of $\mu^{sw} = 1$ and $\mu^{sf} = \mu_0^{sf}$ i.e. $\delta^s = \left(1 - \mu_0^{sf}\right) = \mu_0^{uf}$ we have

$$u^s = \frac{1}{2}\left(\left[\mu_0^{uf} - \eta^s\right] + \sqrt{\left[\mu_0^{uf} - \eta^{ss}\right]^2 + 4\eta^{ss}}\right)$$

$$v^s = u^s - \mu_0^{uf}.\blacksquare$$

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[15]There is another solution which is $u^u = u_0^u = 0$ and $\mu^{sw} = \mu_0^{sw}$ satisfying (2.6). This is unstable since if u_0^u is purturbed away from zero, μ^{sw} fails to converge to μ_0^{uw} and infact it converges to one.

Proof of Lemma 2: Using 2.7 and 2.6 $\mu^{sf} = \mu_0^{sf}$ and $\mu^{sw} = \mu_0^{sw}$ do not change and are given by initial conditions. Using 2.4 and 2.2 we have $\mu_0^{hw} - u^h = \mu_0^{hf} - v^h$ or $v^h = u^h - \left(\mu_0^{hw} - \mu_0^{hf}\right) = u^h - \delta_0^h$, where δ_0^h is the initial mismatch between skilled workers and skilled firms. We can then solve for u^s from equation 2.5 and 2.3, (or 2.4 and 2.2) as before

$$u^h = \frac{1}{2}\left(\left[\delta_0^h - \eta^{hh}\right] + \sqrt{\left[\delta_0^h - \eta^{hh}\right]^2 + 4\eta^{hh}\mu_0^{hw}}\right)$$

$$v^h = u^h - \delta_0^h = v_{10}^h.\blacksquare$$

Proof of Corollary 1: a) Define function

$$u\left(\mu^w\right) = \frac{1}{2}\left(\left[\mu^w - \mu_0^{sf} - \eta\right] + \sqrt{\left[\mu^w - \mu_0^{sf} - \eta\right]^2 + 4\eta^{ss}\mu^w}\right).$$

Note that $u\left(\mu_0^{sw}\right) = u_{00}^s$, $u\left(1\right) = \overline{u}^s$, and $u\left(\mu^w\right) \geq 0$ for $0 \leq \mu^w \leq 1$. Differentiate $u\left(\mu^w\right)$ w.r.t. μ^w and obtain

$$\frac{\partial u\left(\mu^w\right)}{\partial \mu^w} = \frac{1}{2}\left(1 + \frac{1}{2}\frac{2\left[\mu^w - \mu_0^{sf} - \eta\right] + 4\eta^{ss}}{\sqrt{\left[\mu^w - \mu_0^{sf} - \eta\right]^2 + 4\eta^{ss}\mu^w}}\right)$$

$$= \frac{\frac{1}{2}\sqrt{\left[\mu^w - \mu_0^{sf} - \eta\right]^2 + 4\eta^{ss}\mu^w} + \frac{1}{2}\left(\mu^w - \mu_0^{sf}\right) + \eta^{ss}}{\sqrt{\left[\mu^w - \mu_0^{sf} - \eta\right]^2 + 4\eta^{ss}\mu^w}}$$

$$= \frac{u\left(\mu^w\right) + \eta^{ss}}{\sqrt{\left[\mu^w - \mu_0^{sf} - \eta\right]^2 + 4\eta^{ss}\mu^w}} \geq 0$$

Therefore $\overline{u}^s = u\left(1\right) \geq u\left(\mu_0^{sw}\right) = u_{00}^s$. Result follows. b) From previous lemmas. $\blacksquare$

Appendix 2:

A Derivation of Nash Bargaining Wages

To avoid solving very similar equations for the value functions, the following lemma is needed for the derivation of the value functions, Nash bargaining wages, and the derivation of threshold schooling costs.

Lemma 5 : *Define a set of equations in J, V, E, U and ω with parameters $\beta, \theta, p, c_\pi, b, \lambda u, \lambda v$ and σ as*

$$J = p - \omega + \beta \left[(1 - \sigma)J + \sigma V \right]$$

$$V = -c_\pi + \beta \left[\lambda u J + (1 - \lambda u) V \right]$$

$$E = \omega + \beta \left[\sigma U + (1 - \sigma)E \right]$$

$$U = b + \beta \left[\lambda v E + (1 - \lambda v)U \right]$$

$$\omega = \arg\max_{\omega} \theta \log (J - V) + (1 - \theta) \log (E - U)$$

then the solutions are given by:

$$\omega = (1 - \theta)(p + c) + \theta b$$

$$J = \frac{1}{(1 - \beta)} \left[-c_\pi + \frac{(p + c_\pi - b) \theta (1 - \beta [1 - \lambda u])}{(1 - \beta [1 - \sigma - \lambda u])} \right]$$

$$V = \frac{1}{(1 - \beta)} \left[-c_\pi + \frac{(p + c_\pi - b) \theta \beta \lambda u}{(1 - \beta [1 - \sigma - \lambda u])} \right]$$

$$E = \frac{1}{(1 - \beta)} \left[b + \frac{(p + c_\pi - b)(1 - \theta)(1 - \beta [1 - \lambda v])}{(1 - \beta [1 - \sigma - \lambda v])} \right]$$

$$U = \frac{1}{(1 - \beta)} \left[b + \frac{(p + c_\pi - b)(1 - \theta) \beta \lambda v}{(1 - \beta [1 - \sigma - \lambda v])} \right].$$

Proof of Lemma 5: Note that

$$(1 - \beta) J = p - \omega - \beta \sigma \left[J - V \right]$$

$$(1 - \beta) V = -c_\pi + \beta \lambda u \left[J - V \right]$$

$$(1 - \beta) E = \omega - \beta \sigma \left[E - U \right]$$

$$(1 - \beta) U = b + \beta \lambda v \left[E - U \right]$$

then

$$J = \frac{p - \omega}{(1 - \beta)} - \frac{\beta \sigma}{(1 - \beta)} \left[J - V \right] \tag{A5}$$

$$V = \frac{-c_\pi}{(1 - \beta)} + \frac{\beta \lambda u}{(1 - \beta)} \left[J - V \right]$$

$$E = \frac{\omega}{(1 - \beta)} - \frac{\beta \sigma}{(1 - \beta)} \left[E - U \right]$$

$$U = \frac{b}{(1 - \beta)} + \frac{\beta \lambda v}{(1 - \beta)} \left[E - U \right]$$

26

Differencing we get,

$$[J - V] = \frac{p - \omega + c_\pi}{(1 - \beta)} - \frac{\beta (\sigma + \lambda u)}{(1 - \beta)} [J - V]$$

$$[E - U] = \frac{\omega - b}{(1 - \beta)} - \frac{\beta (\sigma + \lambda v)}{(1 - \beta)} [E - U]$$

$$[J - V] \left(1 + \frac{\beta (\sigma + \lambda u)}{(1 - \beta)} \right) = \frac{p - \omega + c_\pi}{(1 - \beta)}$$

$$[E - U] \left(1 + \frac{\beta (\sigma + \lambda v)}{(1 - \beta)} \right) = \frac{\omega - b}{(1 - \beta)}$$

$$[J - V] = \frac{p - \omega + c_\pi}{(1 - \beta [1 - (\sigma + \lambda u)])}$$

$$[E - U] = \frac{\omega - b}{(1 - \beta [1 - (\sigma + \lambda v)])}$$

First order conditions from the wage equations give:

$$\frac{\theta}{p - \omega + c_\pi} = \frac{1 - \theta}{\omega - b} \tag{A6}$$

$$\omega = (1 - \theta) (p + c_\pi) + \theta b$$

Replacing the value of ω we have,

$$[J - V] = \frac{\theta (p - b + c_\pi)}{(1 - \beta [1 - (\sigma + \lambda u)])} \text{ and } [E - U] = \frac{(1 - \theta) (p - b + c_\pi)}{(1 - \beta [1 - (\sigma + \lambda v)])}$$

Therefore, inserting the value differences in (A5) will give the result. ∎

Skilled sector wages:

Proof of Lemma 3: Rewrite the value functions (3.16), (3.17), (3.21), and (3.15) as before. Notice that from Lemma 5, the wages are independent of $\lambda u, \lambda v$ and σ. Therefore the wages in the skilled sector are given by the parameters $p = p^s$ and $c_\pi = \left(c + \xi^f s c_f \right)$. Hence

$$\omega^s = (1 - \theta) \left(p^s + c + \xi^f s c \right) + \theta b. \tag{A7}$$

This proves $\omega^s(1, 0) > \omega^s(0, 1)$. ∎

Unskilled sector wages:

The only relevant state for the unskilled sector is the state of no training, i.e. $\left(\xi^f, \xi^w\right) = (0,0)$. We rewrite (3.18), (3.14), (3.19) and (3.20) as:

$$E^u = \omega^u + \beta\left[\sigma^u U^u + (1 - \sigma^u)E^u\right] \tag{A8}$$

$$U^u(0,0) = b - \beta\left[\lambda^{uu}v_{00}^u E^u + (1 - \lambda^{uu}v_{00}^u)U^u(0,0)\right]$$

$$J^u = p^u - \omega^u + \beta\left[\sigma^u V^u + (1 - \sigma^u)J^u\right] \tag{A9}$$

$$V^u = -c + \beta\left[\lambda^{uu}u_{00}^u J^u + (1 - \lambda^{uu}u_{00}^u)V^u\right] \tag{A10}$$

then using lemma 5 we have,

$$\omega^u(0,0) = (1 - \theta)(p^u + c) + \theta b.$$

$\blacksquare$

B   Skilled Sector Welfare:

Proof of Proposition 4: Notice that in lemma 5, by plugging in the parameters $p = p^s, c_\pi = \left(c + \xi^f sc_f\right)$, $b' = b$, $\sigma = \sigma^s, \lambda v = \lambda^{ss}\overline{v}^s$ and $\lambda u = \lambda^{ss}\overline{u}^s$ we get the equations in the skilled sector as (3.17), (3.16), (3.14), (3.18) and (3.22). We get the skilled sector values when any agent invest as

$$E^s\left(\xi^f, \xi^w\right) = \frac{1}{(1 - \beta)}\left[b + \frac{\left(p^s + c + \xi^f sc_f - b\right)(1 - \theta)\left(1 - \beta\left[1 - \lambda^{ss}\overline{v}^s\right]\right)}{\left(1 - \beta\left[1 - \sigma^s - \lambda^{ss}\overline{v}^s\right]\right)}\right] \tag{A11}$$

$$U^s\left(\xi^f, \xi^w\right) = \frac{1}{(1 - \beta)}\left[b + \frac{\left(p + c + \xi^f sc_f - b\right)(1 - \theta)\beta\lambda^{ss}\overline{v}^s}{\left(1 - \beta\left[1 - \sigma^s - \lambda^{ss}\overline{v}^s\right]\right)}\right] \tag{A12}$$

$$J^s\left(\xi^f, \xi^w\right) = \frac{1}{(1 - \beta)}\left[-\left(c + \xi^f sc_f\right) + \frac{\left(p + c + \xi^f sc_f - b\right)\theta\left(1 - \beta\left[1 - \lambda^{ss}\overline{u}^s\right]\right)}{\left(1 - \beta\left[1 - \sigma^s - \lambda^{ss}\overline{u}^s\right]\right)}\right] \tag{A13}$$

$$V^s\left(\xi^f, \xi^w\right) = \frac{1}{(1 - \beta)}\left[-\left(c + \xi^f sc_f\right) + \frac{\left(p + c + \xi^f sc_f - b\right)\theta\beta\lambda^{ss}\overline{u}^s}{\left(1 - \beta\left[1 - \sigma^s - \lambda^{ss}\overline{u}^s\right]\right)}\right] \tag{A14}$$

therefore

$$E^s(1,0) - E^s(0,1) = \frac{sc_f}{(1-\beta)}\left[\frac{(1-\theta)(1-\beta[1-\lambda^{ss}\overline{v}^s])}{(1-\beta[1-\sigma^s-\lambda^{ss}\overline{v}^s])}\right] > 0 \tag{A15}$$

$$U^s(1,0) - U^s(0,1) = \frac{sc_f}{(1-\beta)}\left[\frac{(1-\theta)\beta\lambda^{ss}\overline{v}^s}{(1-\beta[1-\sigma^s-\lambda^{ss}\overline{v}^s])}\right] > 0 \tag{A16}$$

$$J^s(1,0) - J^s(0,1) = \frac{sc_f}{(1-\beta)}\left[-1 + \frac{\theta(1-\beta[1-\lambda^{ss}\overline{u}^s])}{(1-\beta[1-\sigma^s-\lambda^{ss}\overline{u}^s])}\right] < 0 \tag{A17}$$

$$V^s(1,0) - V^s(0,1) = \frac{sc_f}{(1-\beta)}\left[-1 + \frac{\theta\beta\lambda^{ss}\overline{u}^s}{(1-\beta[1-\sigma^s-\lambda^{ss}\overline{u}^s])}\right] < 0 \tag{A18}$$

In the unskilled sector the only relevant agent is the vacant unskilled firm whose value is $V^s(1,0) = V^s(0,1) = -c$. Since in the state of training there are no unskilled workers are active. ∎

C Derivation of threshold schooling costs

When $(\xi^f,\xi^w) = (0,0)$ from lemma 5 we get:

$$U^u(0,0) = \frac{b}{1-\beta} + \frac{\beta\lambda^{uu}v_{00}^u}{1-\beta}\frac{(1-\theta)(p^u+c-b)}{1-\beta[1-\sigma^u-\lambda^{uu}v_{00}^u]}$$

$$V^s(0,0) = -\frac{c}{1-\beta} + \frac{\beta\lambda^{ss}u_{00}^s}{1-\beta}\frac{\theta(p^s+c-b)}{1-\beta(1-\sigma^s-\lambda^{ss}u_{00}^s)}.$$

When the firm invests, i.e. $(\xi^f,\xi^w) = (1,0)$ from lemma 5 we get:

$$V^s(1,0) = -\frac{c+sc}{1-\beta} + \frac{\beta\lambda^{ss}\overline{u}^s}{1-\beta}\frac{\theta(p^s+c-b+sc)}{1-\beta(1-\sigma^u-\lambda^{ss}\overline{u}^s)}. \tag{A19}$$

When the worker invests, i.e. $(\xi^f,\xi^w) = (0,1)$ from lemma 5 and the previous equations we get:

$$U^u(0,1) = U^s(0,1) - sc. \tag{A20}$$

We calculate the threshold Nash-equilibrium cost for the worker by equating $U^u(0,0) = U^u(0,1)$ respectively. Therefore

$$sc_w^{*n} = \frac{\beta(1-\theta)}{1-\beta}\left[\frac{\lambda^{ss}\overline{v}^s(p^s+c-b)}{1-\beta[1-\sigma^s-\lambda^{ss}\overline{v}^s]} - \frac{\lambda^{uu}v_{00}^u(p^u+c-b)}{1-\beta[1-\sigma^u-\lambda^{uu}v_{00}^u]}\right]. \tag{A21}$$

Obtain, Nash-equilibrium cost for the firm by equating and $V^s(0,0) = V^s(1,0)$, this implies,

$$-sc_f^{*n} + \frac{\theta\left(p^s+c-b+sc_f^{*n}\right)\beta\lambda^{ss}\overline{u}^s}{1-\beta(1-\sigma^u-\lambda^{ss}\overline{u}^s)} = \frac{\theta(p^s+c-b)\beta\lambda^{ss}u_{00}^s}{1-\beta(1-\sigma^s-\lambda^{ss}u_{00}^s)}$$

$$\left[\frac{\theta\beta\lambda^{ss}\overline{u}^s}{1-\beta(1-\sigma^u-\lambda^{ss}\overline{u}^s)} - 1\right]sc_f^{*n} = (p^s+c-b)\left(\frac{\beta\lambda^{ss}\theta u_{00}^s}{1-\beta(1-\sigma^s-\lambda^{ss}u_{00}^s)} - \frac{\beta\lambda^{ss}\theta\overline{u}^s}{1-\beta(1-\sigma^u-\lambda^{ss}\overline{u}^s)}\right)$$

therefore

$$sc_f^{*n} = (p^s + c - b) \frac{(\overline{\alpha} - \alpha_{00})}{(1 - \overline{\alpha})}, \text{ where} \tag{A22}$$

$$\overline{\alpha} = \frac{\beta \lambda^{ss} \theta \overline{u}^s}{1 - \beta (1 - \sigma^s - \lambda^{ss} \overline{u}^s)} \text{ and } \alpha_{00} = \frac{\beta \lambda^{ss} \theta u_{00}^s}{1 - \beta (1 - \sigma^s - \lambda^{ss} u_{00}^s)}.$$

∎

Proof of Proposition 3:

i) Follows from (3).

ii) Let $g(x) = \frac{\beta \lambda^{ss} x}{1 - \beta (1 - \sigma^s - \lambda^s x)}$ then $g'(u) = \frac{\beta \lambda^{ss} (1 - \beta (1 - \sigma^u))}{(1 - \beta (1 - \sigma^u - \lambda^{ss} u))^2} > 0$ is an increasing function in u. Since $\overline{u}^s > u_{00}^s$ and $\overline{\alpha} = \theta g(\overline{u}^s)$ and $\alpha_{00} = \theta g(u_{00}^s)$ therefore $\overline{\alpha} > \alpha_{00}$ and $1 > \overline{\alpha}$. Since $p^s > b - c$, the result follows.

iii) Follows from (3).

iv) Note that from (ii)

$$\frac{\partial sc_f^{*n}(\theta)}{\partial \theta} = \frac{(g(\overline{u}^s) - g(u_{00}^s))(1 - \theta g(\overline{u}^s)) + g(\overline{u}^s) \theta (g(\overline{u}^s) - g(u_{00}^s))}{(1 - \theta g(\overline{u}^s))^2}$$

$$= \frac{(g(\overline{u}^s) - g(u_{00}^s))}{(1 - \theta g(\overline{u}^s))^2} > 0$$

∎

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Tables and Figures

Table 1: Calibrated parameter values

Notation	Value	Source
β	0.99	Shimmer 2005
μ_0^{ws}	0.56	Normalized using Nickell and Bell (1994)
μ_0^{wu}	0.44	Normalized using Nickell and Bell (1994)
u_{00}^s	0.09	Normalized using Nickell and Bell (1994)
u_{00}^u	0.24	Normalized using Nickell and Bell (1994)
v_{00}^s	0.17	Normalized using ONS prior
v_{00}^u	0.16	Normalized using ONS prior
μ_0^{fs}	0.65	Equation (5.33)
μ_0^{fu}	0.35	Equation (5.33)
δ^s	-0.08	$= \mu_0^{sw} - \mu_0^{sf} = -\delta^u$
$\sigma^u = \sigma^s$	0.10	Shimmer (2005)
λ^{ss}	3.01	Equation (5.34)
λ^{uu}	0.50	Equation (5.34)
b	0.4	Shimmer 2005
c	0.21	Shimmer 2005
θ	0.72	Shimmer 2005
p^s	2.12	Acemoglu and Zilibotti (2000)
p^u	1	Normalized

Table 2: Wefare gain from training evaluated at the baseline parameters

π	p^s/p^u	sc_p^{n*}	sc_w^{n*}	sc_f^{n*}	$\frac{W_\pi^f - W_{00}^f}{W_{00}^f}$	$\frac{W_\pi^w - W_{00}^w}{W_{00}^w}$	$\frac{W_\pi^e - W_{00}^e}{W_{00}^e}$	$\frac{W_\pi^s - W_{00}^s}{W_{00}^s}$
01	2.12	34.21	21.73	0.83	0.11	0.03	0.07	0.08
10	2.12	34.21	21.73	0.83	-0.21	0.23	0.03	0.08

Table 3: Wefare gain from Training: Sensitivity Analysis I

π	p^s/p^u	sc_p^{n*}	sc_w^{n*}	sc_f^{n*}	$\frac{W_\pi^f - W_{00}^f}{W_{00}^f}$	$\frac{W_\pi^w - W_{00}^w}{W_{00}^w}$	$\frac{W_\pi^e - W_{00}^e}{W_{00}^e}$	$\frac{W_\pi^s - W_{00}^s}{W_{00}^s}$
01	2.25	40.77	23.81	0.89	0.12	0.04	0.07	0.09
10	2.25	40.77	23.81	0.89	-0.20	0.25	0.04	0.09
01	2.50	53.48	27.86	1.00	0.14	0.05	0.09	0.10
10	2.50	53.48	27.86	1.00	-0.18	0.27	0.06	0.10

Table 4: Wefare gain from Training: Sensitivity Analysis II

π	b	sc_p^{n*}	sc_w^{n*}	sc_f^{n*}	$\frac{W_\pi^f - W_{00}^f}{W_{00}^f}$	$\frac{W_\pi^w - W_{00}^w}{W_{00}^w}$	$\frac{W_\pi^e - W_{00}^e}{W_{00}^e}$	$\frac{W_\pi^s - W_{00}^s}{W_{00}^s}$
01	0.40	34.21	21.73	0.83	0.11	0.03	0.07	0.08
10	0.40	34.21	21.73	0.83	-0.21	0.23	0.03	0.08
01	0.45	35.61	21.51	0.81	0.12	0.03	0.07	0.09
10	0.45	35.61	21.51	0.81	-0.21	0.22	0.04	0.09
01	0.50	37.01	21.29	0.79	0.13	0.03	0.07	0.10
10	0.50	37.01	21.29	0.79	-0.21	0.20	0.04	0.10

Table 5: Skill Premium and Poductivity Ratios

	Skill Premium	
p^s/p^u	$\pi = 0$	$\pi \neq 0$
2.12	1.50	1.00
2.25	1.56	1.00
2.50	1.67	1.00

Table 6: Skill premium and Unemployment Benefits

	Skill Premium	
b	$\pi = 0$	$\pi \neq 0$
0.40	1.50	1.00
0.45	1.47	1.00
0.50	1.45	1.00

Table 7: Job Training and Income Inequality

π	p^s/p^u	gini
00	2.12	0.38
01	2.12	0.40
10	2.12	0.45
00	2.25	0.39
01	2.25	0.41
10	2.25	0.45
00	2.50	0.40
01	2.50	0.42
10	2.50	0.47

Table 8: Job Training and Income Inequality

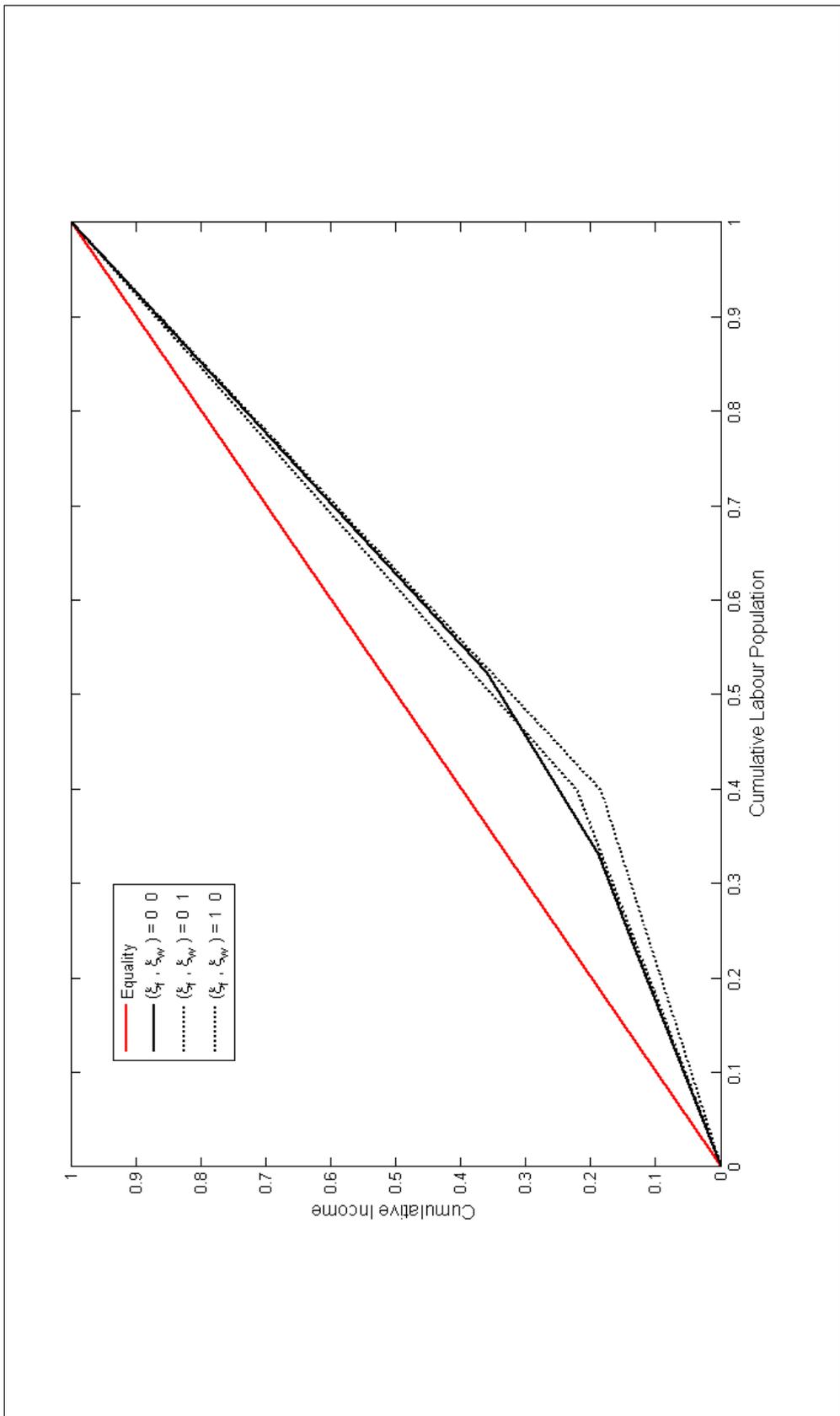| π	b	gini
00	0.40	0.38
01	0.40	0.40
10	0.40	0.45
00	0.45	0.37
01	0.45	0.38
10	0.45	0.43
00	0.50	0.36
01	0.50	0.37
10	0.50	0.41

Figure 3: Lorenz Curve: $b = 0.4$ and $p^s/p^u = 2.121$.