Estimating the Productivity Gains of Importing*

[Preliminary]

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Abstract

Trade in intermediate inputs raises firm productivity as it enables producers to access both better and novel inputs of production. The question is: by how much? This paper develops a framework to answer this question. We explicitly allow for (i) firms sourcing multiple products from multiple countries, (ii) heterogeneity in the quality of these varieties, and (iii) heterogeneity in productivity and fixed costs across firms. We provide direct evidence that all these aspects are empirically important. Our main results are as follows. First, we derive a simple formula that is a consistent estimator for the distribution of productivity gains of past liberalization episodes and can be implemented in readily available firm-level data. In particular, the formula only requires knowledge of firms' domestic expenditure share and the elasticity of substitution between domestic and foreign varieties, which we obtain via production function estimation and exogenous variation in import spending. Secondly, we show how to perform counterfactual policy analysis. With homothetic demand, this is possible using simple linear econometric techniques. For the population of French importers, we find that the median firm gains 11% relative to autarky. A 1% reduction in trade barriers increases firm productivity by 0.3% on average.

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1 Introduction

Understanding the effects of access to foreign inputs on firm productivity is an important task in the field of international economics, especially given the prevalence of input trade in the data. Trade in intermediates can increase firm productivity by providing access to higher quality and novel inputs. A natural question is: what is the magnitude of these productivity gains? Using theory and microdata, this paper provides a simple methodology for estimating the magnitude of these productivity gains at the firm level.

To study the effects of trade in intermediate inputs, we consider a general framework of importing, which is arguably important for applied work. Firms produce using capital, labor and a number of intermediate inputs. Each intermediate can be sourced domestically or from a set of foreign countries. We assume a CES structure between the local variety and a foreign bundle of the imported ones. The foreign varieties of a given input are imperfect substitutes in production and may differ in quality. Thus, import demand arises from complementarities as well as quality differences across inputs. Importing is, however, limited by the presence of fixed costs. We model a firm's sourcing behavior as the solution to a static profit maximization problem. That is, firms choose which products to import and from which countries (the extensive margin), as well as the quantities imported (the intensive margin), to minimize the unit cost of production. To match the richness of the micro-data, we allow for various sources of heterogeneity. At the country level, we assume that each product can be supplied by multiple sourcing countries ("varieties"), which differ in both their prices and qualities. At the firm level we assume that firms can differ in two dimensions: physical productivity and fixed costs. In particular, we allow for firms fixed costs of sourcing to both be correlated with firm productivity and variety quality. Hence, productive firms might for example also be efficient in setting up trading relations (and therefore have low fixed costs of sourcing) or countries, which supply high quality products might at the same time also be easier to access in the sense of having a lower fixed cost requirement. The aim of this paper is to bring this framework to the data to study how changes in the trading environment - e.g. changes in fixed costs or input prices - affect firms' "endogenous" productivity, defined as the amount of output than can be produced with a given level of expenditure in intermediates.

The main result of our paper is to develop a simple and consistent estimator for the productivity gains of importing at the firm level. In particular, we show that knowledge of the change in the firm's domestic expenditure share is sufficient to measure the effect of a shock to the trading environment on the firm's endogenous productivity. More precisely, the change in a firm's endogenous productivity associated with any foreign shock is given by the change in the domestic share of intermediate spending raised to the power of $-\gamma/(\varepsilon - 1)$, where γ is the elasticity of output to intermediates and ε is the elasticity of substitution between domestic and international varieties. This prediction, which is the firm-level analogue of Arkolakis et al. (2012), is very general as it only relies on a CES demand system across domestic and foreign varieties. Surprisingly, this result does neither rely on any assumptions about the underlying distribution of fixed costs, firm productivity and country quality, nor on any assumptions on the market structure on output markets. Intuitively: the static gains from trade at the firm level are fully summarized in firms facing lower input prices for their chosen input bundle. Conditional on a demand system for imported intermediaries, firms' import demand can simply inverted to read off the change in prices. Such change in prices turns out to be precisely given by the change in domestic spending raised to $-\gamma/(\varepsilon-1)$.

An immediate application of this result is that we can measure the firm-level productivity gains from trade directly from observed domestic expenditure shares in the population of firms, as long as we have estimates of the two elasticities γ and ε. In particular, one does not even require detailed micro data on firms' trading behavior as long as one observed the domestic share in material spending. By the same logic, we can study any *past* shock to the trading environment as long as we have data on domestic expenditure shares before and after. The remarkable fact is that we can perform such measurement of the gains without solving for the extensive margin of trade, nor knowledge of any other structural primitives of the model, such as the joint distribution of fixed costs and qualities across countries or the distribution of prices, which firms face. In particular, we also do not have to take a stand on *how* firms end up with their set of trading partners. Hence, our estimate of the gains from trade is consistent regardless of wether firms find their trading partners on a spot market, a process of network formation or through costly search.

While this procedure gives us a simple and consistent estimator for the distribution of productivity gains of importing, there is no direct link to aggregate welfare. This should not come as a surprise, because we did not even specify preferences or the market structure on output markets yet. Hence, if one wanted to study the aggregate implications of this margin of international trade one would need to specify the macro side of the economy and use the estimated distribution of firm-level gains as an input into the assumed interaction of firms on the output side. This decomposition between the estimation of the productivity gains at the firm-level and the macro structure is very useful because it allows our procedure to be applicable in many environments: for a given change in the distribution of domestic expenditure shares, the firm-level gains in physical productivity are fully determined. One can then use additional data (or assumptions) on the nature of product markets to discipline how these micro-gains aggregate into macroeconomic outcomes.

We then turn to the study of *counterfactuals*. While changes in expenditure shares are still sufficient to measure the effect on productivity, we now need more structure from the model to predict how the shares react to a counterfactual shock. In particular, we need a solution to the firm's extensive margin of trade problem which specifies which products are imported and where from[1]. We first show that this problem is in general quite involved, at least as long as one does not impose stringent assumptions on the underlying distribution of quality and fixed costs at the country level. Intuitively, the profit maximization problem of firms is in general non-convex and one would need to evaluate all different sourcing strategies to predict firm behavior. Hence, for our counterfactual analysis we assume that the fixed cost of importing is constant across varieties[2]. Thus, with quality

[1]Note that the method outlined above for the study of past events required only properties from the firm's intensive margin problem, namely the optimal allocation of expenditure between foreign and domestic varieties.

[2]When both fixed costs and qualities are heterogeneous across countries the extensive margin problem is not tractable. With these two dimensions of heterogeneity, the firm needs to evaluate the profit function in all possible combinations of countries to figure out the optimal strategy.

being the only dimension of heterogeneity across countries, the firm's sourcing strategy reduces to a threshold: conditional on importing, the firm sources varieties of sufficiently high quality. We further assume that there is a continuum of countries and that quality is Pareto distributed across countries.

In this context, we show that the effects of shocks to international prices, qualities, or fixed costs on firms' productivity can be measured with knowledge of a single "auxiliary" parameter which captures how foreign combined quality reacts to changes in the number of countries sourced. This elasticity, which we refer to as "returns to scale" parameter, captures the firms' extensive margin adjustment and is therefore all we need to predict changes in endogenous productivity. We show that we can estimate this elasticity from a simple optimality condition that relates domestic expenditure shares with the number of varieties sourced using simple regression analysis. Hence, we can recover the correct structural parameter to counterfactual analysis without having to actually solve the firms' problem. In particular, we do not need to impose assumptions on the distribution of productivity and fixed costs at the firm level - as long as fixed costs are not correlated with country characteristics, we can still estimate the sensitivity of firms' endogenous productivity with respect to counterfactual policy shocks.

We then take this framework to French microdata on the population of importing manufacturing firms. First we estimate the productivity gains from importing relative to autarky. Because expenditure shares are readily available in the data, we only need to estimate the elasticities γ and ε. We estimate these elasticities with a procedure akin to production function estimation. Using optimality conditions from the firm's intensive margin problem only[3], we can express the value of output as a function of labor, capital, expenditure in intermediates, the expenditure share on domestic intermediates and physical productivity. Crucially, the domestic share is a sufficient statistic for the firm's import decisions and captures the endogenous productivity gains from international sourcing. Thus, the domestic share can be treated as an additional input whose output elasticity is given by the trade elasticity ε. We estimate γ and ε with the proxy method in De Loecker and Warzynski (ming), where domestic expenditure shares are instrumented with shocks to world export supply following Hummels et al. (2011).

We find that the productivity gains from importing are small. They are 12% for the average and 5% for the median French firm. The gains are larger for exporters, members of a foreign group and larger firms, which is expected as these firms are likely to either have lower fixed costs or more to gain from the participation in international markets. The reason why the firm-level gains are limited resonates well with the results of Arkolakis et al. (2012): Many importers simply do not import enough for the gains to be substantial. As this number does not rely on any assumptions about the microstructure of trade, *any* model that combines domestic and imported inputs in a CES fashion will predict exactly the same static firm-level gains given the micro-data. Hence, if one thinks that international sourcing leads to higher productivity gains at the firm-level one would need to argue that there are important dynamic gains through e.g. higher innovation incentives or technology adoption. As for counterfactual policy, our results imply that a one percent increase in import prices reduced firm productivity by 0.3%. Roughly half of these losses are due to higher input costs as a

[3]A functional form assumption about the demand faced in output markets by firms is also required.

given input bundle is now more expensive relative to the domestic numeraire. The remaining half is due to firms' endogenous response in that higher import prices change firms' incentives to participate in foreign markets and hence firms drop some marginal trading partners.

The remainder structure of the paper is as follows. In Section 2 we lay out the general framework and derive main result, namely the sufficient statistic for firms' productivity gains of importing. We then turn to the evaluation of counterfactual policy, characterize the solution of firms' extensive margin of importing and provide a simple formula for firms' productivity response to a variety of foreign shocks. Section 3 contains our empirical application using French data. Section 4 concludes.

2 The Basic Framework

We consider a framework of import demand that nests most of the existing models and is arguably the most relevant model of applied empirical work. Importantly, we explicitly allow for firms sourcing their inputs from multiple, heterogeneous import partners and we do not restrict firms' demand system to be homothetic across sourcing partners. More formally, the production structure takes the following structure:

$$\begin{aligned} y &= \varphi f\left(l, k, x_1, .., x_K\right) \\ f\left(x_1, .., x_K\right) &= l^{1-\alpha-\gamma} k^{\alpha}\left(\Pi_{k=1}^{K} x_k^{B_k}\right)^{\gamma} \\ x_k &= \left(x_k^{D\,\frac{\varepsilon-1}{\varepsilon}} + m_k^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \\ x_k^D &= \eta_k\left(q^D, \varphi\right) z_k^D \\ m_k &= \left(\int_{c\in\Sigma_k} \left(\eta_k\left(q_c, \varphi\right) z_{ck}\right)^{\frac{\rho-1}{\rho}} dc\right)^{\frac{\rho}{\rho-1}}. \end{aligned} \tag{1}$$

In words: products, of which there are K many, are combined in a Cobb-Douglas way to produce a final good. Each product is a CES aggregate of a domestic input and a foreign efficiency bundle. Such efficiency bundles are in turn the product of physical units z_{kc} and their firm-specific quality flows $\eta_k(q, \varphi)$, where φ is firm productivity and q is country quality. Hence, q^D is the level of quality of the domestic input. The foreign bundle is a CES aggregate of a continuum of varieties (countries) and Σ_k denotes firms' *sourcing strategy*, i.e. the set of foreign countries product k is sourced from. Σ_k will vary in the cross-section of firms, as we assume that both sourcing individual varieties and products is subject to fixed costs.

Assumption 1. *Countries are heterogeneous in 2 dimensions: quality and fixed costs. In particular we let $G_k(q, f_v)$ be the joint distribution of qualities and fixed costs for product k. Furthermore, we allow firms to be heterogeneous in their fixed costs of sourcing. At this point we do not impose any particular structure.*

Furthermore: we denote prices including any transport costs and allow for high quality goods being more expensive

Assumption 2. *We assume that prices depend on variety quality and parametrize prices as*

$$p_{ck} = \tau p^*_{ck} \equiv \alpha_k q^{\nu}_{ck} \text{ with } \alpha > 0 \text{ and } \nu > 0.$$

Note that $\nu = 1$ denotes the case where price-adjusted qualities (or pure prices) are equalized. We will see: for the first part of this paper, i.e. the identification of the productivity gains from trade, this assumption is innocuous.

Firm-specific quality flows To introduce non-homotheticites, we model quality flows as

$$\tilde{\eta}_{kc}(\varphi) \equiv q_c h_k(q_c, \varphi).$$

The homothetic case is the one where

$$h_k(q_c, \varphi) = 1$$

and quality flows η_{kc} are simply proportional to qualities q_c. Note that it is simply a normalization to set h to unity in the homothetic case - we still have $q_{k,min}$ as the level of the quality distribution. Note also that we can without loss of generality normalize the quality of the domestic bundle to unity, i.e. $q^D \equiv 1$. In general, the function h_k determines wether firm productivity φ and variety quality q are substitutes or complements. In particular, there is quality-productivity complementarity when h is log-supermodular in (q, φ). If h is log-submodular, firm productivity and product quality are substitutes.

Given this setup, firms' import demand is simply the solution to the firms' profit maximization problem. In particular, it is convenient to split the firm's problem into the cost-minimization problem given a sourcing strategy Σ and the choice of the optimal firm size y and sourcing strategy given the cost function. Formally,

$$\pi \equiv \max_{\Sigma,y,l,k} \left\{ py - \Gamma(\Sigma, y, \varphi) - Rk - wl - \sum_{(c,k)\in\Sigma} f_{ck} \right\}, \tag{2}$$

where

$$\Gamma(\Sigma, y, \varphi, l, k) \equiv \min_{z} \left\{ \sum_{(c,k)\in\Sigma} p_{ck} z_{ck} \text{ s.t. } \varphi f(l, k, x_1, .., x_K) \geq y \right\}, \tag{3}$$

is the firm's cost function. Here p denotes the demand function the firm faces, $\Sigma = \left[K, \{\Sigma_k\}_{k\in K}\right]$ is the firm's sourcing strategy , w and R denotes the wage and interest rate, which the firm takes as given and $f(.)$ is the firms' production function given in (1). At this point we do not need to take a stand on the nature of competition or market structure on the output side, i.e. the demand function p is still unrestricted.

The aim of this paper is to take this framework to the data. In particular, we are interested in using the solution to (2) to ask how high are the productivity gains from importing, i.e. how much worse off in terms of productivity would a firm be if we to forced it to operate in autarky (infinite fixed costs). Similarly, we are interested in analyzing firms' responses to policy changes such as a decrease in the fixed costs of sourcing f_v (e.g. due to changes in communication technology or regulation) or a change in variable trade costs τ. Albeit conceptually easy, the answers to these questions are not straightforward. The reason is that - in general - (2) is a hard problem. More precisely, while the cost-minimization problem given firms' extensive margin of trade Σ is extremely tractable, actually solving for the optimal sourcing strategy is difficult unless we impose strong assumptions on the joint distribution between qualities and fixed costs, $G_k(q, f_v)$.

The usual intuition of Melitz-type models of the exporting literature suggests that the extensive margin of import demand should satisfy a sorting condition with respect to firm productivity φ, i.e. not only import status but also the number of products and the number of varieties sourced should be positively correlated with firm productivity so that international sourcing should be hierarchical. This intuition, however, is incorrect. The reason is the interdependence between the different choices on the extensive margin. International sourcing on the input side is a vehicle to reduce the variable cost of production. Hence, a particular variety is imported whenever the reduction in the average production costs outweighs the incurred fixed costs. As long as there is some complementarity across imported varieties, i.e. as long as the production function features some form of "love for variety", these cost reductions depend on the *entire* sourcing strategy Σ. Thus, it might be that unproductive firms source multiple varieties with low fixed costs and low quality flows and high productivity firms concentrate on few fixed cost expensive varieties, which yield high quality flows. This interdependence renders the characterization of the extensive margin of importing much harder than for the case of exports. For exporting firms, the (outward) sourcing strategy can essentially be solved "market by market" - at least as long as production subject to constant returns to scale - see for example Eaton et al. (2011). For imports, however, interdependencies in production are likely to be crucial. The intuition from the exporting literature that firms' extensive margin of trade features a hierarchy has therefore no direct counterpart for firms' importing decisions.[4]

The main insight of this paper is that we do not have to solve for the optimal sourcing strategy to answer a host of questions, which are arguably of major importance. In particular, using standard micro-data we can consistently identify the distribution of firm-level productivity gains of trade relative to autarky as there is a simple observable sufficient statistic for firms' sourcing strategy Σ. The same holds true for any evaluation of past policies which are observed in the data. It is only when we want to conduct counterfactual experiments of policies that have not been observed that we need to solve for the extensive margin problem in (2).

[4] Note, however, that this does not imply that general results concerning the extensive margin cannot be derived. If for example the demand elasticity exceeds unity, more productive firms import and more productive firms adopt a sourcing strategy that leads to lower unit costs, i.e. $\gamma(\Sigma(\varphi')) \leq \gamma(\Sigma(\varphi))$ if $\varphi' > \varphi$. Hence, similar to the exporting intuition, more productive firms sell more and thus have a higher incentive to reduce their marginal costs by incurring the fixed costs of importing additional products/varieties. However, again this does *not* imply that more productive firms source *more* varieties or products.

2.1 Evaluating the Productivity Effects of Observed Trade Policy

Let us first consider the case of observed policy changes. This could either be the comparison between the current allocation and the counterfactual allocation in autarky, or the analysis of a trade shock (i.e. a shock that affects firms' import demand) which occurred in the past. The main result of this section is that we only need to solve the firms' *intensive* margin problem (3) to measure the productivity gains through the lens of the model.

2.1.1 Characterization of the Intensive Margin

Now consider a given firm φ, which sources its products from a set Σ of varieties. Given this set, the fixed costs are irrelevant irrelevant for firms' import demand. In particular, it is easy to see from (1) that firms care only about price-adjusted qualities:

$$\xi_{ck}(\varphi) = \xi_k(q,\varphi) \equiv \frac{\tilde{\eta}_{kc}(\varphi)}{p_{ck}} = \frac{q_c h_k(q_c,\varphi)}{\alpha_k q_c^{\nu}} = \frac{1}{\alpha_k} q_c^{1-\nu} h_k(q_c,\varphi).$$

Note that price-adjusted qualities depend on firm productivity is there are complementarities. Conditional on the sourcing strategy, expenditure shares follow the usual formula:

$$s_{ck}(\Sigma,\varphi,y) = \frac{\xi_{ck}(\varphi)^{\rho-1}}{\int_{c\in\Sigma}\xi_{ck}(\varphi)^{\rho-1}\,dc},$$

where we explicitly denote the dependence on the endogenous sourcing strategy Σ. Letting X_k^I be the total import *spending* on product k we get that

$$\begin{aligned}
\eta_{ck}z_{ck} = \eta_{ck}\frac{z_{ck}p_{ck}}{p_{ck}} &= \frac{\eta_{ck}}{p_{ck}} s_{ck} X_k^I = \xi_{ck}(\varphi)\,\frac{\xi_{ck}(\varphi)^{\rho-1}}{\int_{c\in\Sigma}\xi_{ck}(\varphi)^{\rho-1}\,dc} X_k^I \\
&= \frac{\xi_{ck}(\varphi)^{\rho}}{\int_{c\in\Sigma}\xi_{ck}(\varphi)^{\rho-1}\,dc} X_k^I.
\end{aligned}$$

Hence, import services for product k are given by

$$\begin{aligned}
m_k = \left(\int_{c\in\Sigma_k}(\eta_{ck}z_{ck})^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} &= \frac{1}{\int_{c\in\Sigma}\xi_{ck}(\varphi)^{\rho-1}\,dc} X_k^I \left(\int_{c\in\Sigma_k}\left(\xi_{ck}(\varphi)^{\rho}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \\
&= \left(\int_{c\in\Sigma_k}\xi_{ck}(\varphi)^{\rho-1}\right)^{\frac{1}{\rho-1}} X_k^I \\
&\equiv A_k(\Sigma_k,\varphi)\,X_k^I.
\end{aligned}$$

For further reference, let us define the endogenous import quality A.

Definition 3. The endogenous quality of import expenditure is given by

$$A_k(\Sigma_k,\varphi) \;=\; \left(\int_{c\in\Sigma_k} \xi_{ck}(\varphi)^{\rho-1}\right)^{\frac{1}{\rho-1}}. \tag{4}$$

Given $A_k(\Sigma_k,\varphi)$ we can easily solve the trade-off between domestic and foreign varieties at the firm-level. With domestic prices being normalized to unity, this is the simple problem:

$$\max_{X_k^I, z_k^D} \left(\left(h(1,\varphi)\, z_k^D\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(X_k^I\right)^{\frac{\varepsilon-1}{\varepsilon}} \left(A_k(\Sigma_k,\varphi)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \text{ s.t. } z_k^D + X_k^I = X_k, \tag{5}$$

where X_k is total *spending* on product k. The solution takes the following standard form:

$$\begin{aligned} s_k^F &\equiv \frac{X_k^I}{X_k} = \frac{\left(A_k(\Sigma_k,\varphi)\right)^{\varepsilon-1}}{h(1,\varphi)^{\varepsilon-1} + \left(A_k(\Sigma_k,\varphi)\right)^{\varepsilon-1}} \\ s_k^D &\equiv \frac{X_k^D}{X_k} = \frac{h(1,\varphi)^{\varepsilon-1}}{h(1,\varphi)^{\varepsilon-1} + \left(A_k(\Sigma_k,\varphi)\right)^{\varepsilon-1}}, \end{aligned} \tag{6}$$

so that

$$\begin{aligned} x_k &= \left(\left(X_k^D\right)^{\frac{\varepsilon-1}{\varepsilon}} h(1,\varphi)^{\frac{\varepsilon-1}{\varepsilon}} + \left(X_k^I\right)^{\frac{\varepsilon-1}{\varepsilon}} \left(A_k(\Sigma_k,\varphi)\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \\ &\equiv Q_k(\Sigma_k,\varphi)\, X_k. \end{aligned} \tag{7}$$

Definition 4. The endogenous productivity term of material spending is given by

$$Q_k(\Sigma_k,\varphi) = \left(h(1,\varphi)^{\varepsilon-1} + \left(A_k(\Sigma_k,\varphi)\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}, \tag{8}$$

where $A(\Sigma_k,\varphi)$ is given in (4).

(7) and (8) give us a simple characterization of firms' production function using intermediate *spending* as an input into production. With expenditure across products being given by $X_k = B_k X$, we get that

$$y = \varphi l^{1-\alpha-\gamma} k^{\alpha} \left(\Pi_{k=1}^{K} x_k^{B_k}\right)^{\gamma} = \varphi J\left(\varphi, [\Sigma_k]_{k=1}^{K}\right) \tilde{B}^{\gamma} X^{\gamma} k^{\alpha} l^{1-\alpha-\gamma}, \tag{9}$$

where $\tilde{B} = \Pi_{k=1}^{K} B_k^{B_k}$ is a constant and

$$J\left(\varphi, [\Sigma_k]_{k=1}^{K}\right) = \left(\Pi_{k=1}^{K} Q_k(\Sigma_k,\varphi)^{B_k}\right)^{\gamma}. \tag{10}$$

Here, J is akin to an endogenous productivity term which captures the entirety of the static gains from trade. The reason why we look at the object (9) (instead of (1)) is an empirical one - when looking at the firm-level data we see spending (and not physical inputs) in units of a numeraire. Note that international sourcing increases the efficiency of a dollar spent on intermediate inputs. Importantly: $J\left(\varphi, [\Sigma_k]_{k=1}^{K}\right)$ can be measured directly in the microdata *without* having to specify the microstructure. Using (8) and (6) we get that

$$Q_k\left(\bar{q}_k,\varphi\right)=h\left(1,\varphi\right)\left(s_k^D\left(\Sigma_k,\varphi\right)\right)^{-\frac{1}{\varepsilon-1}}, \tag{11}$$

so that effective firm productivity ϑ is given by

$$\vartheta\equiv\underbrace{\varphi\times\Pi_{k=1}^K h_k\left(1,\varphi\right)^{B_k}}_{\text{Exogenous productivity}}\times\underbrace{\Pi_{k=1}^K\left(s_k^D\left(\Sigma_k,\varphi\right)\right)^{-\frac{B_k\gamma}{\varepsilon-1}}}_{\text{Endogenous productivity gains from trade}}. \tag{12}$$

(12) is a powerful equation as it shows that the *observed* domestic shares s^D are sufficient statistics for the endogenous productivity gains from trade. Put differently: conditional on s^D, neither the extensive margin of trade Σ nor any other underlying structural parameters such as the distribution of import quality q, the distribution of fixed costs or the precise functional form $h\left(q,\varphi\right)$ are required to estimate the endogenous gains from trade at the firm level. Any model that imposes a CES demand system across domestic and international varieties will have the exact the same answer for the implied gains from trade given firm-level data on domestic spending shares.

We view (12) as the firm-level analogue of Arkolakis et al. (2012). Broadly speaking, they show that in an aggregate model the domestic expenditure share at the *country level* is a sufficient statistic for welfare as long as the demand system is of the CES form. (12) states that - at the firm level - firms' spending on domestic intermediaries is a sufficient statistic for "firm welfare", i.e. productivity. In the same vain as consumers gain purchasing power by sourcing cheaper or complementary products abroad, firms can lower the effective price of intermediate purchases by tapping into foreign input markets.

Using (12) there are two exercise we can perform. First of all, we can determine the gains from trade for an importing firm relative to only being active in the domestic economy. These are given by:

$$\frac{\vartheta^{Import}}{\vartheta^{Autary}}=\Pi_{k=1}^K\left(s_k^D\left(\Sigma_k,\varphi\right)\right)^{-\frac{B_k\gamma}{\varepsilon-1}}. \tag{13}$$

With data on domestic expenditures (by product), we can evaluate (13). Importantly, (13) can be measured even without access to trade data as the allocation of spending across imported and domestic inputs is available in any standard firm-level dataset. Similarly, suppose one is interested in analyzing the productivity effects of trade policy, e.g. an episode of trade liberalization. One object of immediate interest is the distribution of *changes* of firm-productivity through the access of better or complementary foreign inputs. Akin to (13), it is easy to see that holding innate productivity φ fixed, the firm-level productivity gains from such policies are given by:

$$\left.\frac{\vartheta^{post}}{\vartheta^{pre}}\right|_{\varphi}=\Pi_{k=1}^K\left(\frac{s_k^D\left(\Sigma_k^{post},\varphi\right)}{s_k^D\left(\Sigma_k^{pre},\varphi\right)}\right)^{-\frac{B_k\gamma}{\varepsilon-1}}. \tag{14}$$

Hence, knowledge of the change in the domestic shares is sufficient to analyze the direct, static consequences of trade reform. Of course: opening up to trade might induce firms' to engage in other productivity enhancing activities like $R\&D$, in which case innate productivity φ would also increase. Such increases in complimentary investments are not encapsulated in (14), which only estimates the direct gains from trade holding productivity fixed.

While (13) and (14) are extremely powerful relationships for applied empirical work, some comments are in order. The first goes back to the analogy with Arkolakis et al. (2012). While the intuition is very similar, we want to stress that (12) cannot immediately aggregated into a welfare measure. This is due to two reasons. The first one is trivial: without having specified the nature of product market competition and consumers' preferences, we do not know how the underlying distribution of effective productivity $G(\vartheta)$ translates into aggregate TFP. If one wants to make such aggregate statements, one needs to fully characterize the macroeconomic environment and derive consumers' welfare as a function of $G(\vartheta)$. With such a mapping in hand, one could then use the micro-data and (13) or (14) to analyze the aggregate TFP consequences of trade reform. There is a second caveat, however, namely the expenditure on fixed costs. With trade being subject to fixed costs, import gains are formally equivalent to a process of "technology adoption" - by paying the fixed costs of sourcing, firms get access to a more productive technology. While (12) indeed measures the implications for firm productivity, a welfare analysis would need to take into account the resources spent in fixed costs (see e.g. Gopinath and Neiman (2012)). This requires us to solve firms' extensive margin problem. This is where we turn to now.

2.2 Evaluating the Productivity Effects of Counterfactual Trade Policy

The results above, especially (14), give us a tractable tool to analyze the trade impact of observed policies. (14) is, however, not helpful to analyze *counterfactual* policies. Intuitively, we need to use the structural model to predict how firms' domestic expenditure share changes as a result of a particular policy. In order to do counterfactuals we therefore need to solve for the extensive margin of trade.

2.2.1 Characterization of the Extensive Margin

While the above analysis used properties from the cost-minimization problem for a given sourcing strategy, the actual choice of the extensive margin Σ is the solution to the profit maximization problem. For tractability reasons, we need to impose additional structure, in particular on the distribution of fixed costs and the nature of product market competition.

Assumption 5. *(Fixed Costs) We assume that firm i faces a fixed cost $f_{p,i}$ to source a product from abroad and an additional fixed cost $f_{v,i}$ to source each international variety of a product. Thus, firms can be heterogeneous in their fixed costs of sourcing, but fixed costs cannot be correlated with variety quality q*

As for the nature of product market competition, we make the standard assumption of isoelastic demand.

Assumption 6. *(Demand) We assume that firm i faces demand $p_i = Dy_i^{-1/\sigma}$, where p_i is the price of firm i and D is a potentially endogenous scalar reflecting aggregate demand and tightness of competition.*

While Assumption 6 is mainly for simplicity, Assumption 5 is essential to make any progress in characterizing the solution to the problem - it allows us to reduce a firm's sourcing strategy from an entire set to a scalar, as Assumption 5 implies that sourcing is hierarchical, i.e. if country c with quality q_c is element of Σ_k so are all countries c' with $q_{c'} > q_c$. Hence, Assumption 5 allows us to go from the sourcing strategy Σ_k to a firm-specific productivity cutoff $\overline{q}_{k,i}$. The productivity index $A_k(\Sigma_k, \varphi)$ (see (4)) is therefore given by

$$A_k(\Sigma_k, \varphi) \quad = A_k(\overline{q}, \varphi) = \quad \frac{1}{\alpha_k} \left(\int_{\overline{q}}^{\infty} \left(q_c^{1-\nu} h_k(q_c, \varphi) \right)^{\rho-1} dG_k(q) \right)^{\frac{1}{\rho-1}}. \tag{15}$$

From (9) it is clear that the marginal costs are constant and given

$$MC = \frac{1}{\varphi \tilde{B} J\left(\varphi, [\overline{q}_k]_{k=1}^K\right) \phi(R, w)}, \tag{16}$$

where $\phi(R, w) = \left(\frac{1-\alpha-\gamma}{w}\right)^{1-\alpha-\gamma} \left(\frac{\alpha}{R}\right)^{\alpha} (\gamma)^{\gamma}$ and $J\left(\varphi, [\overline{q}_k]_{k=1}^K\right)$ is given in (10) and we explicitly denote firms' extensive margin of trade by the product-specific cutoffs $[\overline{q}_k]_{k=1}^K$. With isoelastic demand, firms will set a constant mark-up so that variable profits, i.e. profits before fixed costs are accounted for, are given by

$$\pi^V \quad = \quad (p - MC)\, y = \frac{1}{\sigma} py = \frac{1}{\sigma} D^{\sigma} \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma-1} \left(\frac{1}{MC}\right)^{\sigma-1}.$$

Hence, the extensive margin of importing is determined from

$$\{\bar{q}_k\}_{k=1}^K = \max_{\{\bar{q}_k\}_{k=1}^K} \left\{ \mu(R, w)\, \varphi^{\sigma-1} J\left(\varphi, [\overline{q}_k]_{k=1}^K\right)^{\sigma-1} - \sum_{k=1}^{K} \left\{ f_v\left(1 - G_k(\overline{q}_k)\right) + f_p 1\left[1 - G_k(\overline{q}_k) > 0\right] \right\} \right\}, \tag{17}$$

where

$$\mu(R, w) \equiv \frac{1}{\sigma} D^{\sigma} \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma-1} \left(\phi(R, w)\, \tilde{B}\right)^{\sigma-1}, \tag{18}$$

and $1 - G_k(\overline{q}_k)$ is the measure of countries sourced by the firm. Given the structure of the fixed costs encapsulated in Assumption 5, there are two possible solutions:

1. Either $G_k(\overline{q}_k) = 1$, i.e. $\overline{q}_k = \infty$

2. Or $\overline{q}_k < \infty$ (interior solution).

We can solve the problem in two steps. Conditional on sourcing a product, i.e. paying the fixed cost f_p, the firm will choose the number of varieties optimally. This will imply a particular cutoff

$\overline{q}$.[5] Given this optimal cutoff $\overline{q}$, we then simply compare the two options "sourcing $\overline{q}$ and paying f_p" versus "not sourcing any varieties and not paying f_p".

Let us consider the optimal interior solution for product k, $\overline{q}_k < \infty$. This cutoff solves the optimality condition (note that everything is continuous in the cutoff $\overline{q}_k$)

$$\mu(R,w)\,\varphi^{\sigma-1}J\left(\varphi,[\overline{q}_k]_{k=1}^{K}\right)^{\sigma-1}B_k\gamma(\sigma-1)\frac{Q_k'(\bar{q}_k,\varphi)}{Q_k(\bar{q}_k,\varphi)}+f_v^k g_k(\overline{q}_k)=0, \tag{19}$$

which is simply the necessary condition from (17). In the Appendix we show that (19) can be written as

$$\frac{\rho-1}{\gamma(\sigma-1)}\frac{1}{\mu(R,w)}\frac{1}{\varphi^{\sigma-1}J\left(\varphi,[\overline{q}_k]_{k=1}^{K}\right)^{\sigma-1}}\frac{f_v^k}{B_k}=\frac{(A_k(\bar{q}_k,\varphi))^{\varepsilon-1}}{h(1,\varphi)^{\varepsilon-1}+(A_k(\bar{q}_k,\varphi))^{\varepsilon-1}}\frac{\left[\overline{q}^{1-\nu}h_k(\overline{q},\varphi)\right]^{\rho-1}}{A_k(\overline{q},\varphi)^{\rho-1}}. \tag{20}$$

Note that (20) determines $\overline{q}_k$ as a function of $\left(\varphi,f_v,\overline{q}_{-k}\right)$. As (20) are effectively K conditions (for each cutoff $\overline{q}_k$), the vector of cutoffs $[\overline{q}_k]_k$ conditional on sourcing the product, i.e. the fixed costs f_p being paid, is determined as a function of firm characteristics $\left(\varphi,f_v^k\right)$.

One other convenient property of (20) is that we can easily express it in terms of observable quantities. This can give us a diagnostic on the fit of the model but it will also be a key condition to identify firms' fixed costs in our empirical application. In particular firm sales $S(\varphi)$ are given by

$$\begin{aligned}
S(\varphi) &= p(\varphi)\,y(q)=p(\varphi)^{1-\sigma}D^{\sigma}=D^{\sigma}\left(\frac{\sigma}{\sigma-1}MC(\varphi)\right)^{1-\sigma}\\
&= D^{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(\tilde{B}\phi(R,w)\right)^{\sigma-1}\varphi^{\sigma-1}J\left(\varphi,[\overline{q}_k]_{k=1}^{K}\right)^{\sigma-1}\\
&= \sigma\mu(R,w)\,\varphi^{\sigma-1}J\left(\varphi,[\overline{q}_k]_{k=1}^{K}\right)^{\sigma-1}.
\end{aligned}$$

Similarly, the foreign expenditure share is given by

$$1-s_k^D(\varphi)=\frac{(A_k(\bar{q}_k,\varphi))^{\varepsilon-1}}{h(1,\varphi)^{\varepsilon-1}+(A_k(\bar{q}_k,\varphi))^{\varepsilon-1}}.$$

Finally, the expenditure share on the *marginal* variety is given by

$$s(\overline{q},\varphi,\overline{q})=\frac{\left[\overline{q}^{1-\nu}h_k(\overline{q},\varphi)\right]^{\rho-1}}{\int_{\overline{q}}^{\infty}\left[s^{1-\nu}h_k(s,\varphi)\right]^{\rho-1}dG(s)}=\frac{\left[\overline{q}^{1-\nu}h_k(\overline{q},\varphi)\right]^{\rho-1}}{A_k(\overline{q},\varphi)^{\rho-1}}.$$

Hence, optimal sourcing implies that

$$f_v^k=\frac{\gamma(\sigma-1)}{(\rho-1)\,\sigma}B_kS(\varphi)\left(1-s_k^D(\varphi)\right)s(\overline{q},\varphi,\overline{q}), \tag{21}$$

[5] At this point it might still be the case that conditional on paying f_p, f_v is so high that the firm would still not want to source a particular variety. We will later impose sufficient assumptions for that not to occur. In particular, we will show below that if q is drawn from a Pareto, there will effectively by an Inada-type condition, so that there will always be an interior solution, i.e. $\overline{q} < \infty$.

Note that (21) holds true regardless of the distribution of quality $G_k(q)$ and wether or not import demand is non-homothetic. In particular, we can use (21) to gauge the importance of firm-specific idiosyncrasies embedded in fixed costs. In particular, (21) suggests the regression

$$ln\left(s_k\left(\bar{q},\varphi,\bar{q}\right)\right)=\delta_k-\gamma_1 ln\left(S\left(\varphi\right)\right)-\gamma_2 ln\left(1-s^D\left(\varphi\right)\right)+ln\left(f_v^k\right),$$

i.e. in data format

$$ln\left(s_{k,i}^{last}\right)=\delta_s+\delta_k-\gamma_1 ln\left(S_i\right)-\gamma_2 ln\left(1-s_i^D\right)+u_i, \tag{22}$$

where δ_s and δ_k are sets of product and sector fixed effects (which in theory reflect differences in (B_k,ρ,σ,γ) across sectors. If fixed costs were constant, the theory predicts that $\gamma_1=\gamma_2=1$ and that $var\left(u_i\right)$ should be zero. Hence, the R^2 of (22) tells us something about the need to have firm-specific fixed costs (within industries). Note (22) holds regardless of the specification of homotheticities and/or the Pareto assumption on quality. Also: if $R^2<1$, i.e. fixed costs were actually firm-specifics, we expect the estimate of γ_1 and γ_2 to be biased for two reasons. If f_v was independent of φ, higher fixed costs would both increase the expenditure on the marginal variety and reduce sales endogenously. Hence, the coefficients γ should be biased towards zero. If fixed costs and productivity were itself correlated, a negative correlation would further bias the coefficients towards zero.

With (20) at hand, we can now characterize the entire solution. Let $\bar{q}_k\left(\varphi,f\right)$ be the solution to (20) and

$$\tilde{Q}_k\left(I_k,\varphi,f\right)\equiv I_k Q_k\left(\bar{q}_k\left(\varphi,f\right),\varphi\right)+\left(1-I_k\right),$$

where

$$I_k=\begin{cases}1 & \text{if product } k \text{ is imported}\\ 0 & \text{otherwise}\end{cases}.$$

Hence, $\tilde{Q}_k=Q_k\left(\bar{q}_k\left(\varphi,f\right),\varphi\right)$ if the product is imported (and the firms then optimally choses $\bar{q}_k\left(\varphi,f\right)$) or $\tilde{Q}_k=1$ if the product is only sourced domestically. The optimal sourcing strategy is then given by

$$\left\{I_k\right\}_{k=1}^K=\max_{I_k\in\{0,1\}}\left\{\varphi^{\sigma-1}\left(\Pi_{k=1}^K\tilde{Q}_k\left(I_k,\varphi\right)^{B_k}\right)^{\sigma-1}-\sum_{k=1}^K I_k\left[f_v\left(1-G_k\left(\bar{q}_k\right)\right)+f_p\right]\right\}. \tag{23}$$

This is simply the usual fixed costs, variable costs trade-off between "high" $\tilde{Q}_k\left(I_k,\varphi\right)$ and low f_p. This problem can be solved very quickly computationally. A key property of the solution is the recursive structure of the optimality condition (20), where the other products' cutoffs $\left[\bar{q}_j\right]_{j\neq k}$ only enter the firms' problem via the statistic $J\left(\varphi,\left[\bar{q}_k\right]_{k=1}^K\right)$. We describe the computational procedure in the Appendix.

2.2.2 The Homothetic Case

One particular parametrization of the model, which has a very tractable solution and will likely to remain the workhorse model in applied work is the homothetic case, i.e. the case of $h(q,\varphi) = 1$. To derive even stronger results, let us also assume that country quality q takes the form of a Pareto distribution.

Assumption 7. *(Heterogeneity across countries) Variety quality q is distributed according to a pareto*

$$G_k(q) = \Pr(q_{ck} \leq q) = 1 - \left(\frac{q_{k,\min}}{q}\right)^{\theta}.$$

Hence, the density and the mean quality are

$$g_k(q) = \theta \frac{q_{k,\min}{}^{\theta}}{q^{\theta+1}} \text{ and } E_k[q] = \frac{\theta}{\theta-1} q_{k,min}. \tag{24}$$

Furthermore we need to impose an assumption to make things well defined

$$(\rho-1)(1-\nu) < \theta.$$

Using these assumptions, (4) yields

$$A_k(\overline{q},\varphi) = \left(\int_{c\in\Sigma_k} \xi_{ck}(\varphi)^{\rho-1}\right)^{\frac{1}{\rho-1}} = \frac{1}{\alpha_k}\left(\int_{\overline{q}_k(\varphi)}^{\infty} q^{(1-\nu)(\rho-1)} dG_k(q)\right)^{\frac{1}{\rho-1}}, \tag{25}$$

where the second equality stresses that the sourcing strategy Σ_k is characterized by a firm-specific cutoff $\overline{q}_k(\varphi)$.[6] As G_k is Pareto, we get that

$$A_k(\overline{q},\varphi) = A_k(\overline{q}(\varphi)) = \frac{1}{\alpha_k}\left(q_{k,min}^{\theta} \frac{\theta}{\theta-(1-\nu)(\rho-1)} \overline{q}_k(\varphi)^{(1-\nu)(\rho-1)-\theta}\right)^{\frac{1}{\rho-1}}, \tag{26}$$

where our notation stresses that in the case of homotheticity, firm-specific attributes affect the "price index" of imported varieties only through the extensive margin. Using (26), (8) and (10) it is easy to see that it is only the heterogeneity in firms' extensive margin choices $[\overline{q}_k(\varphi)]_k$, which induces cross-sectional variation in the gains from trade. In the data, (marginal) qualities are not directly observable. Hence, it is useful to express $[\overline{q}_k(\varphi)]_k$ in terms of the number of countries a firm sources from. Clearly, the optimal number of varieties $n_k(\varphi)$ is related to the chosen cutoff $\overline{q}_k(\varphi)$ via

$$n_k(\varphi) = P_k(q \geq \overline{q}_k(\varphi)) = \left(\frac{q_{k,min}}{\overline{q}_k(\varphi)}\right)^{\theta}. \tag{27}$$

[6]This cutoff can also be a function of firm-specific fixed costs.

Substituting into (26) yields

$$
\begin{aligned}
A_k\left(n_k\left(\varphi\right)\right) &= \frac{1}{\alpha_k}\left(q_{min}^{\theta}\frac{\theta}{\theta-\left(1-\nu\right)\left(\rho-1\right)}\left(q_{k,min}n_k\left(\varphi\right)^{-1/\theta}\right)^{\left(1-\nu\right)\left(\rho-1\right)-\theta}\right)^{\frac{1}{\rho-1}} \\
&= \frac{1}{\alpha_k}E\left[q_k\right]^{\left(1-\nu\right)}\left(\frac{\theta-1}{\theta}\right)^{\left(1-\nu\right)}\left(\frac{\theta}{\theta-\left(1-\nu\right)\left(\rho-1\right)}\right)^{\frac{1}{\rho-1}}\left(\frac{1}{n\left(\varphi\right)}\right)^{\frac{\left(1-\nu\right)\left(\rho-1\right)-\theta}{\theta\left(\rho-1\right)}}. \quad (28)
\end{aligned}
$$

Hence, import quality A_k - as a function of the extensive margin of trade n - takes a simple power form

$$
A\left(n\right)=zn^{\eta}, \tag{29}
$$

where "total factor productivity" is given by

$$
z=\frac{1}{\alpha_k}E\left[q\right]^{\left(1-\nu\right)}\left(\frac{\theta-1}{\theta}\right)^{\left(1-\nu\right)}\left(\frac{\theta}{\theta-\left(1-\nu\right)\left(\rho-1\right)}\right)^{\frac{1}{\rho-1}} \tag{30}
$$

and the "returns to scale to international varieties" is

$$
\eta=\frac{\theta-\left(1-\nu\right)\left(\rho-1\right)}{\theta\left(\rho-1\right)}=\frac{1}{\rho-1}-\frac{1-\nu}{\theta}. \tag{31}
$$

Note that $\eta>0$ because of Assumption 7. The structure for $A\left(n\right)$ is extremely useful because it lends itself to a simple interpretation.

Proposition 8. *Consider the import productivity term z and suppose that $\left(\rho-1\right)\left(1-\nu\right)>1$. Then the following is holds:*

1. *Diversity increases import productivity z, as*
   - $z\left(E\left[q\right],\theta,\rho\right)>E\left[q\right]^{1-\nu}=\lim_{\theta\to\infty}z\left(E\left[q\right],\theta,\rho\right)$
   - $\frac{\partial z\left(E\left[q\right],\theta,\rho\right)}{\partial\theta}<0$
2. *Substitutability increases import productivity z, as* $\frac{\partial z\left(E\left[q\right],\theta,\rho\right)}{\partial\rho}>0$
3. *Diversity and substitutability are complements, as* $\frac{\partial^2 z\left(E\left[q\right],\theta,\rho\right)}{\partial\theta\partial\rho}<0$

The intuition for Proposition (8) can be seen by noting that import productivity A satisfies

$$
A^{\rho-1}=\int_{\overline{q}}^{\infty}q^{\left(1-\nu\right)\left(\rho-1\right)}dG\left(q\right)
$$

When $\left(\rho-1\right)\left(1-\nu\right)>1$, the firm is effectively *risk loving* and values diversity. If ρ is high, firms can leverage such quality differences. This is similar to input-output linkages in Jones (2011).

Proposition 9. *The firm's returns to scale to importing,η, satisfies*

$$\begin{aligned}\frac{\partial\eta}{\partial\theta} &= \frac{1}{\rho-1}\frac{(1-\nu)(\rho-1)}{\theta^2} = \frac{(1-\nu)}{\theta^2} > 0 \\ \frac{\partial\eta}{\partial\rho} &= \frac{1}{\theta}\frac{1-(\rho-1)(1-\nu)-(\theta-(\rho-1)(1-\nu))}{(\rho-1)^2} = -\frac{1}{(\rho-1)^2} < 0,\end{aligned}$$

i.e. substitutability and diversity reduce the concavity of the quality schedule.

This result is also intuitive: If quality is very heterogeneous (θ high), the marginal country is of much lower quality relative to the ones, which are already sourced. Hence, import quality flattens out quickly. The same is true for substitutes: As the dispersion of expenditure shares increases in ρ, quality differences are leveraged intensely if ρ is high. Hence, the marginal country, which is by construction also the worst country the firm sources from, receives only a very small expenditure share and hence does not change the total quality of imports very much.

The endogenous productivity function $A(n)$ is a key object in the analysis. Importantly, we can identify the structural parameters (z,η) from readily available trade data at the firm level.

Proposition 10. *Consider the homothetic model above. We can identify the parameters z and η from the auxiliary model from the cross-sectional data on domestic expenditure shares and firms' extensive margin of trade. In particular, domestic expenditure shares are given by*

$$ln(s_D) = -ln\left(1+z^{\varepsilon-1}n^{\eta(\varepsilon-1)}\right), \tag{32}$$

so that

$$ln\left(\frac{1-s_D}{s_D}\right) = (\varepsilon-1)\,ln(z) + \eta(\varepsilon-1)\,ln(n). \tag{33}$$

A log-approximation of (32) *yields*

$$ln(s_{k,D}) = const - (\varepsilon-1)(1-\widehat{s_D})\,\eta ln(n) \tag{34}$$

where $\widehat{s_D}$ is the average domestic share.

Proof. See Appendix. □

Proposition 10 neatly shows how to identify the underlying structural parameters. Given $(\varepsilon, \hat{s}_D)$, the coefficient on $ln(n)$ identifies η. To identify z we then simply use the data on (s_D, n) and match the average domestic expenditure share. Note that we only need to observe the number of international varieties that firms source.

Counterfactual Analysis As explained above, to do counterfactual analysis we need to know how firms' domestic expenditure share changes in response to a shock. Such shock will trigger an adjustment on the extensive and the intensive margins. Hence, we need to know how firms' optimal sourcing strategy n depends on the trading environment. The homothetic model offers an intuitive and extremely tractable way to do so. From the firms' optimality condition (21), we get that

$$f_v^k \frac{\sigma}{\sigma-1}\frac{\rho-1}{\gamma}\frac{1}{B_k} = S(\varphi)\left(1-s_k^D(\varphi)\right)s(\overline{q},\varphi,\overline{q}). \tag{35}$$

The observables $s_k^D(n)$ and $s(\overline{q},\overline{q})$ are related to n and the underlying parameters via

$$s(\overline{q},\overline{q}) = \frac{\theta-(1-\nu)(\rho-1)}{\theta}\frac{1}{n}.$$

Substituting this into the optimality condition for the extensive margin (21), we get that

$$\begin{aligned} f_v^k \frac{\sigma}{\sigma-1}\frac{\rho-1}{\gamma}\frac{1}{B_k} &= \sigma\mu(R,w)\varphi^{\sigma-1}\left(\Pi_{j=1}^K\left(1+z^{(\varepsilon-1)}n_j^{\eta(\varepsilon-1)}\right)^{\frac{B_k\gamma}{\varepsilon-1}}\right)^{(\sigma-1)} \times \\ &\left(\frac{z^{\varepsilon-1}n_k^{\eta(\varepsilon-1)}}{1+z^{\varepsilon-1}n_k^{\eta(\varepsilon-1)}}\right)\frac{\theta-(1-\nu)(\rho-1)}{\theta}\frac{1}{n_k}. \end{aligned}$$

Given the vector (R,w,φ) and the structural parameters, these equations determine the extensive margin of importing $[n_k]_k$. Changes in policy affect firms' extensive margin through

- price adjusted qualities z (the level of prices α_k or the average quality $E[q]$ and transport costs τ)
- fixed costs of sourcing f_v.

To gauge the overall quantitative effect, it is also important to take a stand how much domestic prices (R,w) respond to any of these policy changes under consideration. Given the log-linear structure, this model lends it self to counterfactual analysis quite easily.

Proposition 11. *(Single Product Case) Consider the concreteness the case of $K=1$. Let ϑ be the total endogenous productivity and let φ be constant. Then,*

$$dln(\vartheta) = \lambda(\varphi)\left[dln(z) - \eta dln\left(f_v^k\right) + \eta\left\{\sigma dln(D) - (\sigma-1)\left[(1-\alpha-\gamma)dln(w) + \alpha dln(R)\right]\right\}\right], \tag{36}$$

where $\lambda(\varphi)$ is a firm-specific elasticity given by

$$\lambda(\vartheta) = \frac{\gamma(1-s_D(\varphi))}{1-(\varepsilon-1)\eta\left[\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}\right)(1-s_D(\varphi))+s_D(\varphi)\right]}, \tag{37}$$

and $dln(z)$ denotes the change in quality, prices or trade costs, $dln(f_v)$ is the change in fixed costs, $dln(D)$ is the change in demand and $dln(w)$ and $dln(R)$ is the change in factor prices.

Proof. See Appendix. □

Proposition 11 is very useful for policy analysis as it depends only on the parameters of the "auxiliary model", which can be estimated using simple reduced form methods. If one is interested only in partial equilibrium analysis, we get an even stronger result.

Proposition 12. *Consider the homothetic model above. Suppose you were interested to analyze the effects of counterfactual policies changing the fixed costs of sourcing f, the variables trade costs τ, the level of import prices α_k or the average price-adjusted quality $\frac{q}{p}$ in partial equilibrium, i.e. holding the vector (w, R, D) constant. The policies only depend on the observed domestic shares s_D and the structural parameters $(\sigma, \gamma, \varepsilon, \eta)$, as*

$$dln\left(\vartheta\right) = \lambda\left(\varphi\right)\left[dln\left(z\right) - \eta dln\left(f_v^k\right)\right].$$

In particular, we do not need to identify the distribution of qualities (θ), the degree of complementarities (ρ), the distribution of fixed costs (f_v), the elasticity of prices with respect to quality (ν) or the average import quality (q_{min}).

Proposition 12 is very powerful: given some value of σ, we can estimate $(\gamma, \varepsilon, \eta)$ easily using firm-level data. With just these parameters, we can gauge the productivity effects of a broad variety of foreign shocks, taking into account firms' endogenous response on the extensive margin of trade.

Decomposing Productivity Gains While the above allowed us to (a) estimate the productivity effects of *observed* trade episodes and (b) *counterfactual* trade episodes, we are not in the position yet to decompose such productivity gains. As seen from (29), (31) and (30) there are three margins, which could account for the observed productivity gains

- gains from variety ρ
- gains from diversity θ
- average quality $E\left[q_k\right]$

To decompose the observed gains, we therefore need to identify the parameter vector $(\rho, \theta, E\left[q_k\right], \nu)$. In the homothetic model, we can get these structural parameters from simple reduced form methods. We gather these results in the following Propositions.

Proposition 13. *Consider the homothetic model above. We can identify the structural parameters $(\theta, \rho, \nu, E\left[q_k\right])$ using data on import prices p, the pattern of expenditure across sourcing countries and the estimates (z, η) from above. In particular, average import prices are related to firms' number of varieties by*

$$ln\left(p\left(n\left(\varphi\right)\right)\right) = const\ - \frac{\nu}{\theta} ln\left(n\left(\varphi\right)\right). \tag{38}$$

The within-firm expenditure share on the top κ percent of varieties is given by

$$\theta = \frac{\left(1-\nu\right)\left(\rho - 1\right)}{\frac{ln\left(s_k^{10}\left(\varphi_i, \bar{\xi}_{ik}\right)\right)}{ln(10)} + 1}. \tag{39}$$

Proof. See Appendix. □

Propositions (10) and (13) suggest a tractable procedure to identify the underlying structural parameters.

1. In a first step, identify z and η from the observed domestic spending
2. Using (30), (31), (38) and (39) we then have 4 equations in the 4 underlying unknowns.

	Quantiles				
	25%	50%	75%	90%	99%
No of annual trade interactions per country	10	53	328	3,763	69,371
No of annual trade interactions per product	8	25	74	179	752
No of annual trade interactions per firm	2	8	25	59	238
Value of firm-product-country interactions (1000EUR)	44.15	268.53	1,258.83	5,319.10	70,993.13
Number of firm-product-country observations			705,316		

Notes: See Section **??** in the Appendix for a description of the data.

Table 1: The Concentration of French Imports

3 Empirical Implementation

We now take this framework to data on French manufacturing firms. We proceed in two steps. We first use our sufficiency result about firms' static gains from trade to estimate the distribution of productivity gains relative to autarky in the population of French importers. As firms' domestic shares are directly observed, we only need to estimate the "trade elasticity" ε (Arkolakis et al., 2012), which here is simply the elasticity of substitution between imported and domestic varieties. We estimate ε using an instrumental variables strategy given an estimate of firms' effective productivity ϑ. We then turn to the study of counterfactuals as well as the decomposition of the observed gains. As seen above, we can estimate the required parameters using simple reduced form methods which have a tight structural interpretation.

3.1 Data and Descriptive Statistics

In this section we provide a general overview of the dataset. A detailed description of how the data is constructed is contained in the Appendix. Because we are interested in the demand for inputs, we restrict the analysis to manufacturing firms. We observe import flows for every manufacturing firm in France from the official custom files. Manufacturing firms account for 31% of the population of French importing firms and 56% of total import value in 2001. Overall, French firms trade with a total 226 countries. The flows are classified at the 8-digit (NC8) level of aggregation, which means that the product space consists of roughly 9,500 products. Using unique firm identifiers we can match this dataset to fiscal files, which contain detailed information on firm characteristics. The final sample consists of an unbalanced panel of roughly 260,000 firms which are active between 2001 and 2006.[7]

As the existing literature did not focus on the variety margin of firm-level trade data, we summarize the distribution of variety flows, i.e. product-country cells, across firms in Table 1 .[8] In total we observe roughly 700,000 variety-firm pairs. Given that there are about 30,000 importers in our data, the average importer imports about 23 varieties of potentially different products. Table 1 however

[7]This dataset is not new and has been used in the literature before (e.g. Mayer et al. (2010); Eaton et al. (2004, 2011)). However, these contributions focused almost exclusively on the export side.

[8]Table 1 is of a similar flavor as the discussion of sparsity by Armenter and Koren (2008) but it is slightly different. Whereas they analyze the data on the *flow* level, we aggregate the data within a *firm-variety cell*, because it is these quantities which our model can speak to.

shows that this average is not too informative as international activity is highly concentrated, both geographically (row one) and in the product space (row two). The median country is only active in 50 firm-product cells, whereas the top two exporting countries to France, namely Germany and Italy, report 70,000 interactions in distinct firm-product cells. Similarly, for half of the potential products, i.e. roughly 5,000 products, only 25 country-firm interactions are observed, while the most popular products are shipped into France in more than 750 distinct country-firm combinations.[9] Finally, the two remaining rows confirm the findings of Gopinath and Neiman (2012) for the case of Argentina that imports are also very concentrated at the firm level. While the median firm sources only 8 varieties a year internationally, the top one percent of firms (that is, 300 firms) import 240 varieties. Similarly, while the most active firm-variety pairs are worth more than 70m EUR, a quarter of French importers import less than 45,000EUR worth of the varieties within a year. Thus, at whatever dimension we look, the world of imported inputs is a small world. Few firms are actively participating, and when they do, they tend to import from a small set of countries and only a small subset of potential commodities.

3.2 Estimating the Productivity Gains from Importing

We now estimate the productivity gains from importing relative to autarky for the population of importing manufacturing firms in France. According to (13), these are given by

$$\Pi_{k=1}^{K}\left(s_k^D\left(\Sigma_k,\varphi\right)\right)^{-\frac{B_k\gamma}{\varepsilon-1}}.$$

As we do not observe domestic expenditure shares at the product level, we let $s_k^D = s^D$ so that effective firm productivity is given by

$$\vartheta = \varphi \times s^D\left(\Sigma,\varphi\right)^{-\frac{\gamma}{\varepsilon-1}}.$$

As the expenditure shares are observed, we only need to estimate γ and ε. Combining the assumption of isoelastic demand in product markets (Assumption 6) with the representation of the production function embedded in (9), we arrive at the following expression for firm sales[10]:

$$S = py \propto y^{\frac{\sigma-1}{\sigma}} = \vartheta^{\frac{\sigma-1}{\sigma}} X^{\tilde{\gamma}} k^{\tilde{\alpha}} l^{\tilde{\rho}},$$

where $\tilde{\gamma} = \frac{\sigma-1}{\sigma}\gamma$, $\tilde{\alpha} = \frac{\sigma-1}{\sigma}\alpha$ and $\tilde{\rho} = \frac{\sigma-1}{\sigma}\left(1-\alpha-\gamma\right)$ and S denotes firm sales.

Taking logs,

$$ln\left(S\right) = \delta + \tilde{\alpha} ln\left(k\right) + \tilde{\rho} ln\left(l\right) + \tilde{\gamma} ln\left(X\right) + ln\left(\omega\right), \tag{40}$$

[9]Note in particular the still very large difference between the 99% and 90%-quantile.

[10]Because output in physical units is not observed, we use Assumption 6 to express the production function in terms of sales.

where

$$\begin{aligned} ln(\omega) &\equiv \frac{\sigma-1}{\sigma} ln(\vartheta) = \frac{\sigma-1}{\sigma}\left[-\frac{\gamma}{\varepsilon-1} ln\left(s^D\right) + ln(\varphi)\right] \\ &= -\frac{1}{\varepsilon-1}\tilde{\gamma} ln\left(s^D\right) + \frac{\sigma-1}{\sigma} ln(\varphi). \end{aligned} \tag{41}$$

We estimate equations (40) and (41) using the following two-step procedure. First, we estimate equation (40) using the usual proxy methods for production function estimation. In particular, we follow the procedure in De Loecker and Warzynski (ming) to arrive at estimates of the vector of coefficients $(\tilde{\alpha}, \tilde{\gamma}, \tilde{\rho})$ and an estimate of $ln(\omega)$ for each firm. Second, using the estimated coefficient $\tilde{\gamma}$ and values for $ln(\omega)$, we estimate ε from equation (41). We cannot estimate (41) via OLS, as the required orthogonality restriction clearly fails: s^D is not orthogonal to innate productivity φ. In particular, more productive firms are likely to sort into more and different sourcing countries and this variation in the extensive margin of trade (which is correlated with productivity) will induce variation in firm-specific price indices and hence domestic shares. Hence, we estimate ε from (41) using an instrumentals variable strategy. In particular, we follow Hummels et al. (2011) and instrument s^D with shocks to world export supplies. Through the lens of the model, these are product-specific shocks to import prices French firms face. Using this source of variation, which is orthogonal to innate firm productivity φ, we can identify ε and hence estimate the firm-specific gains from trade:

$$ln\left(\frac{\vartheta}{\varphi}\right) = -\frac{\gamma}{\varepsilon-1} ln\left(s^D(\Sigma,\varphi)\right) = -\frac{\sigma}{\sigma-1}\frac{\tilde{\gamma}}{\varepsilon-1} ln\left(s^D(\Sigma,\varphi)\right), \tag{42}$$

which is essentially a scaled version of the fitted values of (41).

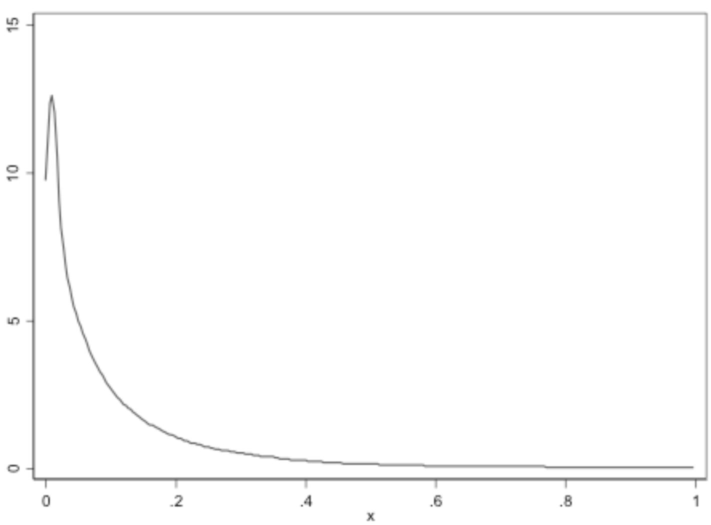
The results of this exercise are reported below. Table 9 in the Appendix reports the results of estimating (40) within all 2-digit sectors in the economy. Hence, we allow for sector-specific output elasticities. Using the estimated coefficients $\tilde{\gamma}_s$, we then build the scaled domestic shares $\tilde{\gamma}_s ln\left(s^D\right)$ and estimate ε from (41) using the instrument. The results of this exercise are contained in Table 2 below. The implied gains from trade are depicted in Figure 1 and summarized in Table 3. We find that, for the average firm, productivity in the observed trade equilibrium is 12% higher than in autarky. For the median firm the productivity gains from trade is 5%.

As explained above, Figure 1 displays the distribution of the static gains from trade at the firm-level. The crucial assumption is the CES structure between domestic and foreign varieties. Conditional on the micro-data (i.e. the domestic expenditure shares) and values of ε and γ, this distribution is consistent under quite general assumptions. In particular, we did not assume anything about the distribution of fixed costs, the underlying heterogeneity on the country level or firms' import demand structure, i.e. whether firm productivity and country quality are complements or substitutes. We also want to stress that the necessity to even estimate productivity $ln(\omega)$ and to use our instrumental variables in (41), only arises because we aimed to estimate γ and ε. If one had outside information about these values, one could estimate the distribution of gains simply from the observed domestic expenditure share s^D. Hence, our procedure is applicable in a variety of settings

	OLS	First stage	IV	IV (2001 shares)	First stage (Diffs)	IV (Diffs)
	$ln(\hat{\omega})$	$\gamma_s \times ln(s_D)$	$ln(\hat{\omega})$	$ln(\hat{\omega})$	$\Delta\gamma_s \times ln(s_D)$	$\Delta ln(\hat{\omega})$
World Export Suppy		-0.014***				
		(0.000)				
$\gamma_s \times ln(s_D)$	-0.096***		-0.499***	-0.525***		
	(0.003)		(0.036)	(0.047)		
ΔWorld Export Suppy					-0.010***	
					(0.003)	
$\Delta\gamma_s \times ln(s_D)$						-0.741**
						(0.356)
N	103,333	103,333	103,333	81,819	67,696	67,696
R^2	0.78	0.11	0.74	0.73	0.00	

Notes: Robust standard errors in parentheses with ***, ** and * respectively denoting significance at the 1%, 5% and 10% levels.

Table 2: Estimating the trade elasticity



Notes: The Figure displays a kernel density estimator based on the empirical distribution of $\frac{1}{\varepsilon-1}log(s_{D,i})$. Data is from year 2004. The value of ε is taken from the third column of Table 2.

Figure 1: Distribution of Productivity Gains

Mean	Median	p25	p75	N
1.1266	1.0511	1.0138	1.1404	113,011

Notes: We are using the industry-specific ε for this table.

Table 3: Distribution of Gains: Moments

	Gains	Gains	Gains	Gains	Gains	Gains	Gains
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Sales	0.015***						
	(0.001)						
log VA		0.009***					
		(0.001)					
Exporter			0.051***				
			(0.002)				
Foreign Group				0.133***		0.079***	
				(0.003)		(0.003)	
log Num Varieties					0.146***	0.135***	
					(0.002)	(0.002)	
log Age							-0.004***
							(0.001)
Observations	113,011	111,722	113,011	113,011	113,011	113,011	108,224
R^2	0.12	0.12	0.12	0.14	0.18	0.19	0.12

Notes: Robust standard errors in parentheses with ***, ** and * respectively denoting significance at the 1%, 5% and 10% levels.

Table 4: Variation in the gains from trade

and provides an easy to implement tool to quantify the gains from importing without having to rely on any substantial assumptions besides the CES production structure.

Having the endogenous gains from trade (42) at hand, we can start to learn about some of the models' underlying structure. In particular, we will project the endogenous gains on different firm characteristics, i.e. run regressions of the form

$$-\frac{\sigma}{\sigma-1}\frac{\tilde{\gamma}}{\varepsilon-1}ln\left(s_{it}^{D}\right)=\delta_s+\delta_t+\beta x_{it}+u_{it}, \tag{43}$$

where δ are sector and time fixed effects and x_{it} are different firm characteristics. To interpret β, consider the homothetic model so that

$$-ln\left(s_{it}^{D}\right)=ln\left(1+A\left(\Sigma\right)^{\varepsilon-1}\right),$$

i.e. firm-characteristics *only* matter via the extensive margin of trade. As A is "increasing" in the extensive margin of trade in that a larger set Σ implies a higher productivity of the import bundle A, we can interpret the partial correlations in (43) as reflecting the equilibrium relationship between different firm characteristics and the accompanying sourcing strategy. The results are contained in Table 4.

3.3 Evaluating Counterfactuals

We now turn to the study of counterfactuals. As shown in Proposition 11 above, a crucial object for counterfactuals is the firm-specific "productivity elasticity", $\lambda\left(\vartheta\right)$, which measures the sensitivity of

	(1) Common ε	(2) Industry-specific ε
z	0.555***	0.627***
	(0.001)	(0.001)
η	0.327***	0.148***
	(0.002)	(0.002)
N	114,723	96,343
R^2	0.88	0.87

Table 5: Estimating η via NLS

firm productivity to changes in trade costs. This elasticity is given by

$$\lambda\left(\vartheta\right)=\frac{\gamma\left(1-s_D\left(\varphi\right)\right)}{1-\left(\varepsilon-1\right)\eta\left[\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}\right)\left(1-s_D\left(\varphi\right)\right)+s_D\left(\varphi\right)\right]}. \tag{44}$$

Given an estimate of γ and a given value of σ, this structural elasticity only depends on the returns to scale parameter η. We now turn to the estimation of such parameter.

First, as argued above, we can estimate η from a simple log-linear relationship between firms' domestic shares and their extensive margin of trade. In particular, Proposition 10 showed that

$$ln\left(s_{D,i}\right)=\text{const}-\left(\varepsilon-1\right)\eta\left(1-\widehat{s_D}\right)ln\left(n_i\right) \tag{45}$$

where $\widehat{s_D}$ is the average domestic share. Estimating (45) in the micro-data allowing for sector and time fixed effects yields a coefficient estimate of -0.296 (with an associated standard error of 0.004). Using $\varepsilon\approx3$ from Table 2 above and an average domestic share of 0.70, the estimated coefficient implies $\hat{\eta}=0.48$.

Instead of working with a log-linear approximation, we can also estimate η (along with z) directly from the non-linear relation between domestic shares and the number of varieties implied by the model:

$$s^D=\frac{1}{1+z^{(\varepsilon-1)}n^{\eta(\varepsilon-1)}} \tag{46}$$

Table 5 contains the results of estimating (46) via non-linear least squares.

Given our estimates of η, we can use (44) to evaluate the productivity elasticity. Given the observed distribution of domestic shares s^D, (44) implies that for the average firm, i.e. a firm with a domestic share of 70%, the elasticity λ is given by

$$\overline{\lambda}=0.35,$$

i.e. a one-percent decline in import prices increases the productivity of importers by 0.35%.

(19)	(20)	(21)	(22)
			Unit Values
Unit Values	Unit Values	Unit Values	nace 4 digit
no Ind Dum	nace 2 digit	nace 4 digit	>1 variety
-0.033***	-0.032***	-0.032***	-0.025***
(0.003)	(0.003)	(0.002)	(0.005)
-0.043***	-0.041***	-0.041***	-0.033***
(0.003)	(0.003)	(0.002)	(0.005)
-0.045***	-0.044***	-0.044***	-0.030***
(0.003)	(0.003)	(0.002)	(0.005)
-0.034***	-0.034***	-0.035***	-0.015***
(0.003)	(0.003)	(0.003)	(0.005)
0.031***	0.008***	0.000	-0.034***
(0.002)	(0.002)	(0.002)	(0.004)
2.714***	2.302***	2.107***	2.126***
(0.002)	(0.016)	(0.043)	(0.079)
2,104,533	2,104,533	2,104,533	486,782
0.65	0.67	0.68	0.75

Table 6: Estimating ν

3.4 Decomposing Gains from Trade

Now consider the decomposition of the gains from trade. In particular we are interested to see wether most of the gains are due to complementarity across foreign inputs or due to quality effects in that foreign inputs are relatively cheap given the quality they offer. As explained above, this requires to to estimate the parameter vector (θ, ν, z). Again, this is extremely transparent in the homothetic model. Given the expression for domestic shares, we can estimate z using NLLS from the moment condition

$$\hat{z} = \arg\min_{z} \sum_{i=1}^{N} \left[s_{D,i} - \frac{1}{1 + z^{\varepsilon-1} n_i^{\eta(\varepsilon-1)}} \right]^2 .$$

Then we use the variation in prices and the extensive margin of trade to identify ν. In particular, Proposition 13 implies that

$$ln\left(p\left(n\left(\varphi\right)\right)\right) = \text{const } - \frac{\nu}{\theta} ln\left(n\left(\varphi\right)\right).$$

The results are contained in Table 6. Finally, we can use these results and (30), (31) and (39) to solve for the parameters. The results are contained in Table 7.

Given these parameters we consider two exercises, which are instructive about the different margins of the gains from trade. We first ask what we as researchers would have concluded had we looked at this data though the lens of a "homogenous quality" world. In particular, conditional on the same

ρ	θ	ν	$E[q]$
1.91	1.35	0.045	0.68

Table 7: Structural parameters

ρ	θ	ν	$E[q]$
$\frac{1+\eta}{\eta} = 3.8$	∞	not identified	$\hat{z}^{\varepsilon-1} = 0.55$

Table 8: Structural parameters: homogenous world

micro-data, we would have estimated the same η and z but then erroneously set θ to infinity by discarding the data on the concentration of the within-product expenditure patterns. The results of this exercise are contained in Table 8 .

4 Conclusion

Firms engage in outsourcing, i.e. in acquiring production inputs from abroad, to increase productivity. This paper developed a framework to estimate the magnitude of these gains. Our main result showed that firms' domestic expenditure share in material spending is a sufficient statistic for these productivity gains. Remarkably, we showed that this statistic is robust under a variety of assumptions. In particular, we did not need to impose any assumptions on the underlying heterogeneity at the firm-level (e.g. innate productivity or the fixed costs of sourcing) or at the country level (e.g. quality, prices or variable trade costs). We also did not need to take a stand on the market structure on output markets. Using micro data from the population of French importers we showed that the productivity gains from importing are small. The median importer gains only 5% of productivity by being allowed to source inputs internationally. In order for importers to generate bigger productivity gains there have to be dynamic gains due to innovation incentives, learning or technology adoption.

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Sector	l	k	m
NACE 14	0.482	0.276	0.259
NACE 15	0.389	0.087	0.561
NACE 17	0.614	0.060	0.351
NACE 18	0.773	0.043	0.320
NACE 19	0.586	0.079	0.342
NACE 20	0.392	0.060	0.522
NACE 21	0.468	0.099	0.424
NACE 22	0.886	-0.002	0.200
NACE 24	0.380	0.080	0.580
NACE 25	0.410	0.085	0.477
NACE 26	0.866	0.021	0.281
NACE 27	0.297	0.049	0.605
NACE 28	0.510	0.099	0.363
NACE 29	0.641	0.045	0.352
NACE 30	0.615	0.073	0.385
NACE 31	0.520	0.058	0.430
NACE 32	0.672	0.059	0.288
NACE 33	0.591	0.078	0.325
NACE 34	0.483	0.095	0.445
NACE 35	0.911	0.036	0.196
NACE 36	0.580	0.057	0.393
NACE 37	0.411	0.181	0.457

Table 9: Production Function Coefficient Estimates, by 2-digit Sector

5 Appendix

5.1 Estimating the production functions

In Table below we report the results of estimating (40) using the procedure of De Loecker and Warzynski (ming).

5.2 Normalizing the average price to unity

Prices are given by

$$p(q) = \alpha q^{1-\nu}.$$

If q is pareto with $q_{k,min}$ and θ, prices are pareto with minimum $\alpha q_{k,min}^{\nu}$ and shape $\frac{\theta}{\nu}$. Hence,

$$E[p] = \frac{\frac{\theta}{\nu}}{\frac{\theta}{\nu}-1}\alpha q_{k,min}^{\nu} = \frac{\theta}{\theta-\nu}\alpha q_{k,min}^{\nu}.$$

Hence, the required normalization is

$$\alpha = \frac{\theta-\nu}{\theta}\frac{1}{q_{k,min}^{\nu}}.$$

5.3 Proof of (20)

The first order condition is given in (19) as

$$\mu(R,w)\,\varphi^{\sigma-1}J\left(\varphi,[\bar{q}_k]_{k=1}^{K}\right)^{\sigma-1}B_k\gamma(\sigma-1)\frac{Q_k'(\bar{q}_k,\varphi)}{Q_k(\bar{q}_k,\varphi)}+f_v^k g_k(\bar{q}_k)=0. \tag{47}$$

Using the definition of $Q(.)$ we get that

$$\frac{Q_k'(\bar{q}_k,\varphi)}{Q_k(\bar{q}_k,\varphi)}=-\frac{\left(\frac{A_k(\bar{q}_k,\varphi)}{\alpha_k}\right)^{\varepsilon-1}}{h(1,\varphi)^{\varepsilon-1}+\left(\frac{A_k(\bar{q}_k,\varphi)}{\alpha_k}\right)^{\varepsilon-1}}\frac{1}{\rho-1}\frac{\left(\left[\bar{q}^{1-\nu}h_k(\bar{q},\varphi)\right]^{\rho-1}g(\bar{q})\right)}{A_k(\bar{q},\varphi)^{\rho-1}}.$$

Using (8) and (15) we get that

$$\begin{aligned}
\frac{Q_k'(\bar{q}_k,\varphi)}{Q_k(\bar{q}_k,\varphi)} &= \frac{\partial ln\left(Q_k(\bar{q}_k,\varphi)\right)}{\partial\bar{q}_k}\\
&= \frac{1}{\varepsilon-1}\frac{\partial ln\left(h(1,\varphi)^{\varepsilon-1}+\left(A_k(\bar{q}_k,\varphi)\right)^{\varepsilon-1}\right)}{\partial\bar{q}_k}\\
&= \frac{1}{\varepsilon-1}\frac{1}{h(1,\varphi)^{\varepsilon-1}+\left(A_k(\bar{q}_k,\varphi)\right)^{\varepsilon-1}}(\varepsilon-1)\left(A_k(\bar{q}_k,\varphi)\right)^{\varepsilon-1}\frac{A_k'(\bar{q}_k,\varphi)}{A_k(\bar{q}_k,\varphi)}\\
&= \frac{\left(A_k(\bar{q}_k,\varphi)\right)^{\varepsilon-1}}{h(1,\varphi)^{\varepsilon-1}+\left(A_k(\bar{q}_k,\varphi)\right)^{\varepsilon-1}}\frac{A_k'(\bar{q}_k,\varphi)}{A_k(\bar{q}_k,\varphi)}.
\end{aligned}$$

But $A_k(\bar{q}_k,\varphi)$ is given in (15) as

$$A_k(\bar{q},\varphi)=\frac{1}{\alpha_k}\left(\int_{\bar{q}}^{\infty}\left[s_c^{1-\nu}h_k(s_c,\varphi)\right]^{\rho-1}dG(s)\right)^{\frac{1}{\rho-1}}.$$

Hence,

$$\begin{aligned}
\frac{A_k'(\bar{q}_k,\varphi)}{A_k(\bar{q}_k,\varphi)} &= \frac{\partial ln\left(A_k(\bar{q}_k,\varphi)\right)}{\partial\bar{q}_k}\\
&= \frac{1}{\rho-1}\frac{\partial ln\left(\left(\int_{\bar{q}}^{\infty}\left[s_c^{1-\nu}h_k(s_c,\varphi)\right]^{\rho-1}dG(s)\right)\right)}{\partial\bar{q}_k}\\
&= -\frac{1}{\rho-1}\frac{\left[\bar{q}^{1-\nu}h_k(\bar{q},\varphi)\right]^{\rho-1}}{\int_{\bar{q}}^{\infty}\left[s_c^{1-\nu}h_k(s_c,\varphi)\right]^{\rho-1}dG(s)}g(\bar{q}).
\end{aligned}$$

Substituting into (47) and rearranging terms yields

$$\mu(R,w)\,\varphi^{\sigma-1}J\left(\varphi,[\bar{q}_k]_{k=1}^{K}\right)^{\sigma-1}B_k\frac{\gamma(\sigma-1)}{\rho-1}\frac{\left(A_k(\bar{q}_k,\varphi)\right)^{\varepsilon-1}}{h(1,\varphi)^{\varepsilon-1}+\left(A_k(\bar{q}_k,\varphi)\right)^{\varepsilon-1}}\frac{\left[\bar{q}^{1-\nu}h_k(\bar{q},\varphi)\right]^{\rho-1}}{\int_{\bar{q}}^{\infty}\left[s_c^{1-\nu}h_k(s_c,\varphi)\right]^{\rho-1}dG(s)}=f_v^k,$$

which can be written as

$$\frac{(A_k(\bar{q}_k,\varphi))^{\varepsilon-1}}{h(1,\varphi)^{\varepsilon-1}+(A_k(\bar{q}_k,\varphi))^{\varepsilon-1}}\frac{\left[\overline{q}^{1-\nu}h_k(\overline{q},\varphi)\right]^{\rho-1}}{\int_{\overline{q}}^{\infty}\left[s_c^{1-\nu}h_k(s_c,\varphi)\right]^{\rho-1}dG(s)}=\frac{\rho-1}{\gamma(\sigma-1)}\frac{1}{\mu(R,w)}\frac{1}{\varphi^{\sigma-1}J\left(\varphi,[\overline{q}_k]_{k=1}^{K}\right)^{\sigma-1}}\frac{f_v^k}{B_k}.$$

As q is pareto, we get from (24) that $g_k(q)=\theta\frac{q_{k,\min}{}^{\theta}}{q^{\theta+1}}$. Hence,

$$\frac{A_k'(\bar{q}_k,\varphi)}{A_k(\bar{q}_k,\varphi)} = -\frac{1}{\rho-1}\frac{\left[\overline{q}^{1-\nu}h_k(\overline{q},\varphi)\right]^{\rho-1}\theta q_{k,\min}^{\theta}\overline{q}^{-(\theta+1)}}{A_k(\overline{q},\varphi)^{\rho-1}}.$$

Substituting into (19), we get

$$\begin{aligned}
f\times\theta q_{k,\min}^{\theta}\overline{q}_k^{-(\theta+1)} &= -\varphi^{\sigma-1}\left(\Pi_{k=1}^{K}Q_k(\bar{q}_k,\varphi)^{B_k(\sigma-1)}\right)B_k(\sigma-1)\frac{Q_k'(\bar{q}_k,\varphi)}{Q_k(\bar{q}_k,\varphi)}\\
&= -\varphi^{\sigma-1}\left(\Pi_{k=1}^{K}Q_k(\bar{q}_k,\varphi)^{B_k(\sigma-1)}\right)B_k(\sigma-1)\frac{\left(\frac{A_k(\bar{q}_k,\varphi)}{\alpha_k}\right)^{\varepsilon-1}}{h(1,\varphi)^{\varepsilon-1}+\left(\frac{A_k(\bar{q}_k,\varphi)}{\alpha_k}\right)^{\varepsilon-1}}\left(-\frac{1}{\rho-1}\frac{\left[\overline{q}^{1-\nu}h_k(\overline{q},\right.}{A}\right.\\
&= \varphi^{\sigma-1}\left(\Pi_{k=1}^{K}Q_k(\bar{q}_k,\varphi)^{B_k(\sigma-1)}\right)B_k(\sigma-1)\frac{\left(\frac{A_k(\bar{q}_k,\varphi)}{\alpha_k}\right)^{\varepsilon-1}}{h(1,\varphi)^{\varepsilon-1}+\left(\frac{A_k(\bar{q}_k,\varphi)}{\alpha_k}\right)^{\varepsilon-1}}\frac{1}{\rho-1}\frac{\left[\overline{q}^{1-\nu}h_k(\overline{q},\varphi)\right]^{\rho-}}{A_k(\overline{q},\varphi}
\end{aligned}$$

Hence,

$$f = \varphi^{\sigma-1}\left(\Pi_{k=1}^{K}Q_k(\bar{q}_k,\varphi)^{B_k(\sigma-1)}\right)B_k\left(\frac{\sigma-1}{\rho-1}\right)\frac{\left(\frac{A_k(\bar{q}_k,\varphi)}{\alpha_k}\right)^{\varepsilon-1}}{h(1,\varphi)^{\varepsilon-1}+\left(\frac{A_k(\bar{q}_k,\varphi)}{\alpha_k}\right)^{\varepsilon-1}}\frac{\left[\overline{q}^{1-\nu}h_k(\overline{q},\varphi)\right]^{\rho-1}}{A_k(\overline{q},\varphi)^{\rho-1}}$$

Rearranging terms where (again see (4))

$$A_k(\bar{q}_k,\varphi)=\left(\int_{\overline{q}}^{\infty}\left[s_c^{1-\nu}h_k(s_c,\varphi)\right]^{\rho-1}dG(s)\right)^{\frac{1}{\rho-1}}.$$

(20) is the optimality condition for the cutoff $\overline{q}_k$.

5.4 Proof of (26)

Now consider

$$\int_{\overline{q}}^{\infty}u^{(1-\nu)(\rho-1)}dG(u)=(1-G(\overline{q}))\int_{\overline{q}}^{\infty}u^{(1-\nu)(\rho-1)}\frac{dG(u)}{1-G(\overline{q})}.$$

As qualities are pareto (see (24)) we get

$$1-G(\overline{q})=\left(\frac{q_{k,min}}{\overline{q}}\right)^{\theta}.$$

Also: the conditional distribution of $q \geq \overline{q}$ is pareto on $\overline{q}$ with shape θ. And

$$\begin{aligned} P\left(q^{(1-\nu)(\rho-1)} \leq z\right) &= P\left(q \leq z^{\frac{1}{(1-\nu)(\rho-1)}}\right) \\ &= 1-\left(\frac{\overline{q}}{z^{\frac{1}{(1-\nu)(\rho-1)}}}\right)^{\theta} \\ &= 1-\left(\frac{\overline{q}^{(1-\nu)(\rho-1)}}{z}\right)^{\frac{\theta}{(1-\nu)(\rho-1)}}. \end{aligned}$$

Hence, $q^{(1-\nu)(\rho-1)}$ is pareto on $\overline{q}^{(1-\nu)(\rho-1)}$ with shape $\frac{\theta}{(1-\nu)(\rho-1)}$. Hence,

$$\int_{\overline{q}}^{\infty} u^{(1-\nu)(\rho-1)} \frac{dG(u)}{1-G(\overline{q})} = \frac{\frac{\theta}{(1-\nu)(\rho-1)}}{\frac{\theta}{(1-\nu)(\rho-1)}-1}\overline{q}^{(1-\nu)(\rho-1)}$$

and

$$\int_{\overline{q}}^{\infty} u^{(1-\nu)(\rho-1)} dG(u) = q_{k,min}^{\theta} \frac{\theta}{\theta-(1-\nu)(\rho-1)} \overline{q}^{(1-\nu)(\rho-1)-\theta}$$

5.5 Proof of Proposition (10)

Average Prices Look at the average price

$$\begin{aligned} p(\overline{q}) &= \int_{\overline{q}}^{\infty} p(q)\, s(q)\, dG(q) \\ &= \alpha \int_{\overline{q}}^{\infty} q^{\nu} \left(\frac{\left(\frac{1}{\alpha_k} q^{1-\nu}\right)^{\rho-1}}{\int_{\overline{q}}^{\infty}\left(\frac{1}{\alpha_k} q^{1-\nu}\right)^{\rho-1} dG(q)}\right) dG(q) \\ &= \alpha \left(\frac{\int_{\overline{q}}^{\infty} q^{(1-\nu)(\rho-1)+\nu} dG(q)}{\int_{\overline{q}}^{\infty} q^{(1-\nu)(\rho-1)} dG(q)}\right) \\ &= \alpha \left(\frac{\int_{\overline{q}}^{\infty} q^{(1-\nu)(\rho-1)+\nu} \frac{dG(q)}{1-G(\overline{q})}}{\int_{\overline{q}}^{\infty} q^{(1-\nu)(\rho-1)} \frac{dG(q)}{1-G(\overline{q})}}\right). \end{aligned}$$

Hence, the numerator and the denominator is the expected value of a RV x^b where x is pareto on $\overline{q}$ with shape θ. But x^b is pareto on $\overline{q}^b$ with shape $\frac{\theta}{b}$. HEnce

$$\begin{aligned} p(\overline{q}) &= \alpha \frac{\frac{\frac{\theta}{(1-\nu)(\rho-1)+\nu}}{\frac{\theta}{(1-\nu)(\rho-1)+\nu}-1}\overline{q}^{(1-\nu)(\rho-1)+\nu}}{\frac{\frac{\theta}{(1-\nu)(\rho-1)}}{\frac{\theta}{(1-\nu)(\rho-1)}-1}\overline{q}^{(1-\nu)(\rho-1)}} \\ &= \alpha \frac{\frac{\theta}{\theta-(1-\nu)(\rho-1)+\nu}}{\frac{\theta}{\theta-(1-\nu)(\rho-1)}}\overline{q}^{\nu} \\ &= \alpha \frac{\theta-(1-\nu)(\rho-1)}{\theta-(1-\nu)(\rho-1)+\nu}\overline{q}^{\nu}. \end{aligned}$$

Now

$$n = \left(\frac{q_{min}}{\overline{q}}\right)^{\theta},$$

so that

$$\overline{q}^{\nu} = \left(\frac{1}{n}\right)^{\frac{\nu}{\theta}} q_{min}^{\nu}.$$

Hence

$$ln\left(p\left(n\left(\varphi\right)\right)\right) = \text{const } - \frac{\nu}{\theta} ln\left(n\left(\varphi\right)\right).$$

Concentration of expenditure shares Now consider the case of θ. For each firm we have now calculated the cutoff $\overline{q}_k\left(\varphi\right)$. Hence, the quality of countries it actually imports from is pareto on $\left[\overline{q}_k\left(\varphi\right), \infty\right]$ with shape θ. Expenditure shares are increasing in q. Hence, the top 10% countries are the countries with $q > q_{ik}^{10}$, where q_{ik}^{10} satisfies

$$0.1 = P\left[q > q_{ik}^{10}\left(\varphi\right)\right] = \left(\frac{\overline{q}_k\left(\varphi\right)}{q_{ik}^{10}\left(\varphi\right)}\right)^{\theta}.$$

Hence,

$$q_k^{10}\left(\varphi\right) = \left(\frac{1}{0.1}\right)^{1/\theta} \overline{q}_k\left(\varphi\right) = 10^{1/\theta}\overline{q}_k\left(\varphi\right). \tag{48}$$

Total expenditure on these top 10% of countries is given by

$$\begin{aligned} s_k^{10}\left(\varphi\right) &= \frac{\int_{q_k^{10}(\varphi)}^{\infty}\left[q^{1-\nu}h\left(q,\varphi\right)\right]^{\rho-1} dG_k\left(q\right)}{\int_{\overline{q}_k(\varphi)}^{\infty}\left[q^{1-\nu}h\left(q,\varphi\right)\right]^{\rho-1} dG_k\left(q\right)} \\ &= \left(\frac{A_k\left(q_k^{10}\left(\varphi\right),\varphi\right)}{A_k\left(\overline{q}_k\left(\varphi\right),\varphi\right)}\right)^{\rho-1}. \end{aligned}$$

The average top-10 spending share at the firm level is that

$$s^{10}\left(\varphi\right) = \sum_{k=1}^{K} s_k^{10}\left(\varphi\right) \frac{1\left(N_k > 0\right) B_k}{\sum_{r=1}^{K} 1\left(N_r > 0\right) B_r}$$

As a moment we target

$$E\left[s_k^{10}\left(\varphi\right)\right] = \int_{\overline{\varphi}}^{\infty} s_k^{10}\left(\varphi\right) \frac{f\left(\varphi\right)}{1 - F\left(\overline{\varphi}\right)} d\varphi,$$

where $\overline{\varphi}$ is the productivity cutoff of importing. In the homothetic case we get the closed form. Using that from (26) we get that

$$A\left(\hat{q},\varphi\right) = A\left(\hat{q}\right) = \left(q_{k,min}^{\theta} \frac{\theta}{\theta - \left(1-\nu\right)\left(\rho-1\right)} \hat{q}^{(1-\nu)(\rho-1)-\theta}\right)^{\frac{1}{\rho-1}}$$

Hence, (48) implies that

$$s_k^{10}\left(\varphi_i, \overline{\xi}_{ik}\right) = \left(\frac{q_k^{10}(\varphi)}{\overline{q}_k(\varphi)}\right)^{(1-\nu)(\rho-1)-\theta} = 10^{\frac{(1-\nu)(\rho-1)-\theta}{\theta}}.$$

Hence, θ is hardwired to ρ as

$$\begin{aligned}\frac{ln\left(s_k^{10}\left(\varphi_i, \overline{\xi}_{ik}\right)\right)}{ln(10)} &= \frac{(1-\nu)(\rho-1)-\theta}{\theta} = \frac{(1-\nu)(\rho-1)}{\theta} - 1 \\ \theta &= \frac{(1-\nu)(\rho-1)}{\frac{ln\left(s_k^{10}\left(\varphi_i, \overline{\xi}_{ik}\right)\right)}{ln(10)} + 1}.\end{aligned}$$

Domestic shares

$$\begin{aligned} ln(s_D(\varphi)) &= -(\varepsilon-1)\, ln\left[Q(n(\varphi))\right] \\ &\approx -(\varepsilon-1)\left[h(\hat{n}) + \eta\beta(\hat{n})\, ln(n(\varphi))\right] \\ &= -(\varepsilon-1)\, h(\hat{n}) - (\varepsilon-1)\,\eta\beta(\hat{n})\, ln(n(\varphi)),\end{aligned}$$

we can run the regression

$$ln\left(s_D^i\right) = \tau + \vartheta ln(n_i) + u_i$$

and get

$$\vartheta = -(\varepsilon-1)\,\eta\left[1 - s_D(\hat{n})\right]. \tag{49}$$

5.6 Proof of Proposition 11

Start with

$$\begin{aligned} f_v^k \frac{\sigma}{\sigma-1}\frac{\rho-1}{\gamma}\frac{1}{B_k} &= \sigma\mu(R,w)\,\varphi^{\sigma-1}\left(1+z^{(\varepsilon-1)}n^{\eta(\varepsilon-1)}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}}\left(\frac{z^{\varepsilon-1}n^{\eta(\varepsilon-1)}}{1+z^{\varepsilon-1}n^{\eta(\varepsilon-1)}}\right)\frac{\theta-(1-\nu)(\rho-1)}{\theta}\frac{1}{n_k} \\ &= \sigma\mu(R,w)\,\varphi^{\sigma-1}\left(\left(1+z^{(\varepsilon-1)}n_j^{\eta(\varepsilon-1)}\right)\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1}\left(z^{\varepsilon-1}n^{\eta(\varepsilon-1)-1}\right)\frac{\theta-(1-\nu)(\rho-1)}{\theta}.\end{aligned}$$

Taking logs and first differences:

$$dln\left(f_v^k\right) = dln(\mu(R,w)) + dln\left(\left(1+(zn^\eta)^{\varepsilon-1}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1} z^{\varepsilon-1}n^{\eta(\varepsilon-1)-1}\right)$$

Also

$$dln\left(\left(1+(zn^\eta)^{\varepsilon-1}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1} z^{\varepsilon-1}n_k^{\eta(\varepsilon-1)-1}\right) = dln\left(\left(1+(zn^\eta)^{\varepsilon-1}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1}\right) + (\varepsilon-1)\, dln(z) + \left[(\varepsilon-1)\eta-1\right] dln\,(n$$

And

$$
\begin{aligned}
dln\left(\left(1+(zn^{\eta})^{\varepsilon-1}\right)^{\frac{\gamma(\sigma-1)}{\varepsilon-1}-1}\right) &= \left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)dln\left(1+(zn^{\eta})^{\varepsilon-1}\right) \\
&= \left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)\frac{d\left[(zn^{\eta})^{\varepsilon-1}\right]}{1+(zn^{\eta})^{\varepsilon-1}} \\
&= \left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)\frac{(zn^{\eta})^{\varepsilon-1}}{1+(zn^{\eta})^{\varepsilon-1}}\frac{d\left[(zn^{\eta})^{\varepsilon-1}\right]}{(zn^{\eta})^{\varepsilon-1}} \\
&= \left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)\frac{(zn^{\eta})^{\varepsilon-1}}{1+(zn^{\eta})^{\varepsilon-1}}dln\left[(zn^{\eta})^{\varepsilon-1}\right] \\
&= \left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)(\varepsilon-1)\frac{(zn^{\eta})^{\varepsilon-1}}{1+(zn^{\eta})^{\varepsilon-1}}\left[dln(z)+\eta dln(n)\right].
\end{aligned}
$$

Hence,

$$
\begin{aligned}
dln\left(f_v^k\right) &= dln(\mu(R,w))+\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)(\varepsilon-1)\frac{(zn^{\eta})^{\varepsilon-1}}{1+(zn^{\eta})^{\varepsilon-1}}\left[dln(z)+\eta dln(n)\right]+(\varepsilon-1)dln(z)+[(\varepsilon-1) \\
&= dln(\mu(R,w))+(\varepsilon-1)\left[\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)\frac{(zn^{\eta})^{\varepsilon-1}}{1+(zn^{\eta})^{\varepsilon-1}}+1\right]dln(z)+\left[(\varepsilon-1)\eta\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)\frac{(z}{1+} \right. \\
&= dln(\mu(R,w))+(\varepsilon-1)\left[\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)(1-s_D(\varphi))+1\right]dln(z)+\left[(\varepsilon-1)\eta\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)(1- \right.
\end{aligned}
$$

so that

$$
dln(n)=\frac{dln\left(f_v^k\right)-dln(\mu(R,w))}{(\varepsilon-1)\eta\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)(1-s_D(\varphi))+(\varepsilon-1)\eta-1}-\frac{(\varepsilon-1)\left(\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)(1-s_D(\varphi))+1\right)}{(\varepsilon-1)\eta\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)(1-s_D(\varphi))+(\varepsilon-1)\eta-1}dln
$$

Now: firm productivity is given by

$$
\vartheta=\varphi\times\left(s^D(n,\varphi)\right)^{-\frac{\gamma}{\varepsilon-1}}=\varphi\times\left(1+(zn^{\eta})^{\varepsilon-1}\right)^{\frac{\gamma}{\varepsilon-1}}.
$$

hence

$$
\begin{aligned}
dln(\vartheta) &= \frac{\gamma}{\varepsilon-1}dln\left(1+(zn^{\eta})^{\varepsilon-1}\right) \\
&= \gamma\frac{(zn^{\eta})^{\varepsilon-1}}{1+(zn^{\eta})^{\varepsilon-1}}\left[dln(z)+\eta dln(n)\right] \\
&= \gamma(1-s_D(\varphi))dln(z)+\gamma(1-s_D(\varphi))\eta dln(n).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
dln\left(\vartheta\right) &= \gamma\left(1-s_D\left(\varphi\right)\right)dln\left(z\right)+\gamma\left(1-s_D\left(\varphi\right)\right)\eta\left[\frac{dln\left(f_v^k\right)-dln\left(\mu\left(R,w\right)\right)}{\left(\varepsilon-1\right)\eta\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)\left(1-s_D\left(\varphi\right)\right)+\left(\varepsilon-1\right)\eta-1}-\frac{(\varepsilon\ \cdots}{(\varepsilon-1)\eta\ \cdots}\right. \\
&= \gamma\left(1-s_D\left(\varphi\right)\right)\left\{\left(1-\frac{\eta\left(\varepsilon-1\right)\left(\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)\left(1-s_D\left(\varphi\right)\right)+1\right)}{\left(\varepsilon-1\right)\eta\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)\left(1-s_D\left(\varphi\right)\right)+\left(\varepsilon-1\right)\eta-1}\right)dln\left(z\right)+\frac{\eta\left(dln\left(\cdots\right.\right.}{\left(\varepsilon-1\right)\eta\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-\cdots\right.}\right. \\
&= \gamma\left(1-s_D\left(\varphi\right)\right)\left\{\left(\frac{-1}{\left(\varepsilon-1\right)\eta\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)\left(1-s_D\left(\varphi\right)\right)+\left(\varepsilon-1\right)\eta-1}\right)dln\left(z\right)+\frac{\eta\left(dln\left(f_v^k\right)\cdots\right.}{\left(\varepsilon-1\right)\eta\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)\cdots}\right. \\
&= \gamma\left(1-s_D\left(\varphi\right)\right)\left\{\left(\frac{1}{1-\left(\varepsilon-1\right)\eta\left[\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)\left(1-s_D\left(\varphi\right)\right)+1\right]}\right)dln\left(z\right)-\frac{\eta\left(dln\left(f_v^k\right)-dln\left(\mu\cdots\right.\right.}{1-\left(\varepsilon-1\right)\eta\left[\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)\cdots\right.}\right. \\
&= \frac{\gamma\left(1-s_D\left(\varphi\right)\right)}{1-\left(\varepsilon-1\right)\eta\left[\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}\right)\left(1-s_D\left(\varphi\right)\right)+s_D\left(\varphi\right)\right]}\left\{dln\left(z\right)-\eta\left(dln\left(f_v^k\right)-dln\left(\mu\left(R,w\right)\right)\right)\right\}
\end{aligned}
$$

Now note that

$$
\begin{aligned}
\mu\left(R,w\right) &= \frac{1}{\sigma}D^{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(\phi\left(R,w\right)\tilde{B}\right)^{\sigma-1} \\
&= \frac{1}{\sigma}D^{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\tilde{B}^{\sigma-1}\left(\left(\frac{1-\alpha-\gamma}{w}\right)^{1-\alpha-\gamma}\left(\frac{\alpha}{R}\right)^{\alpha}\left(\gamma\right)^{\gamma}\right)^{\sigma-1}.
\end{aligned}
$$

Hence,

$$dln\left(\mu\left(R,w\right)\right)=\sigma dln\left(D\right)-\left(\sigma-1\right)\left[\left(1-\alpha-\gamma\right)dln\left(w\right)+\alpha dln\left(R\right)\right]$$

so that

$$dln\left(\vartheta\right)=\lambda\left(\vartheta\right)\left[dln\left(z\right)+\eta\left\{\sigma dln\left(D\right)-dln\left(f_v^k\right)-\left(\sigma-1\right)\left[\left(1-\alpha-\gamma\right)dln\left(w\right)+\alpha dln\left(R\right)\right]\right\}\right],$$

where

$$\lambda\left(\vartheta\right)=\frac{\gamma\left(1-s_D\left(\varphi\right)\right)}{1-\left(\varepsilon-1\right)\eta\left[\left(\frac{\gamma(\sigma-1)}{\varepsilon-1}-1\right)\left(1-s_D\left(\varphi\right)\right)+1\right]}$$

5.7 Decomposing the parameters

The basic equations to solve for $(\theta, \nu, \rho, E[q])$ are given by

$$
\begin{aligned}
z &= \frac{1}{\alpha_k} E[q]^{(1-\nu)} \left(\frac{\theta-1}{\theta}\right)^{(1-\nu)} \left(\frac{\theta}{\theta-(1-\nu)(\rho-1)}\right)^{\frac{1}{\rho-1}} && (50) \\
\eta &= \frac{\theta-(1-\nu)(\rho-1)}{\theta(\rho-1)} = \frac{1}{\rho-1} - \frac{1-\nu}{\theta} && (51) \\
ln(p(n(\varphi))) &= \text{const} \ - \frac{\nu}{\theta} ln(n(\varphi)) && (52) \\
\theta &= \frac{(1-\nu)(\rho-1)}{\frac{ln\left(s_k^{10}\left(\varphi_i, \overline{\xi}_{ik}\right)\right)}{ln(10)} + 1}. && (53)
\end{aligned}
$$

In the data we find that

$$
\begin{aligned}
\widehat{z^{\varepsilon-1}} &= 0.3 \\
\hat{\eta} &= 0.3 \\
\widehat{\nu/\theta} &= 0.034 \\
s_k^{10}\left(\varphi_i, \overline{\xi}_{ik}\right) &= 0.55.
\end{aligned}
$$

In particular

$$
z^{\frac{1}{1-\nu}} \left(\frac{\theta}{\theta-1}\right) \left(\frac{\theta-(1-\nu)(\rho-1)}{\theta}\right)^{\frac{1}{(\rho-1)(1-\nu)}} = \left(\frac{E[q]}{\alpha_k^{\frac{1}{1-\nu}}}\right).
$$