

Reorganization or Liquidation: Bankruptcy Choice and Firm Dynamics*

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Abstract

In this paper, we ask how bankruptcy law affects the financial decisions of corporations and its implications for firm dynamics. According to current US law, firms have two bankruptcy options: Chapter 7 liquidation and Chapter 11 reorganization. Under Chapter 7, the proceeds from liquidating corporate assets are used to repay debt and bankruptcy costs and limited liability exempts corporate shareholders from the excess losses. When firms reorganize under Chapter 11, bankruptcy law determines the negotiation rules over the fraction of debt that is repaid, the reorganized corporation retains its assets and continues to operate. We extend the basic firm dynamics model with endogenous entry and exit and costly equity finance to include both bankruptcy options. We evaluate a bankruptcy policy change recommended by the American Bankruptcy Institute and find that changes to the law can have significant consequences for borrowing costs, capital structure, firm dynamics, and welfare.

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1 Introduction

According to (p.524) Aghion, Hart, and Moore [4] Western bankruptcy procedures “are thought either to cause the liquidation of healthy firms (as in Chapter 7 of the U.S. Bankruptcy Code) or to be inefficient and biased toward reorganization under incumbent management (as in Chapter 11 in the United States).” They add there is no “consensus about how to improve these procedures.” To evaluate the implications of bankruptcy procedures for firm value and industry dynamics, we propose a structural corporate finance model which includes both options (as in the data) and consider a counterfactual policy based on a new proposal by the American Bankruptcy Institute.

Under US bankruptcy law, firms have two bankruptcy options: Chapter 7 liquidation and Chapter 11 reorganization. Under Chapter 7, the proceeds of liquidating the corporation’s assets are used to repay debt and cover bankruptcy costs. Limited liability exempts corporate shareholders from liability for the corporation’s debt beyond liquidation value. When firms reorganize under Chapter 11, on the other hand, bankruptcy law determines the negotiation rules over the fraction of debt that is repaid, the reorganized corporation retains its assets and continues to operate. A simple way to characterize differences in the current law is that absolute priority rule (where debt is senior to equity) is applied in Chapter 7 while it is not applied in Chapter 11. A simple way to characterize the Aghion, Hart and Moore proposal is that it makes absolute priority rule apply in all bankruptcy.

There are several studies which document heterogeneity among firms which choose Chapter 7 and Chapter 11 bankruptcy. One of the more recent papers is by Bris, Welch and Zhu [8] who provide a comprehensive study of the costs of Chapter 7 versus Chapter 11 in a sample of 300 public and private firms in Arizona and New York from 1995-2001. Reorganization by Chapter 11 comprises 80% of their sample. Chapter 11 firms are substantially larger in terms of assets, have a larger fraction of secured debt, and have roughly similar debt to asset ratios to Chapter 7 (see their Table 1). Importantly, their paper documents substantial differences in recovery rates. In particular, Table 13 documents the median (mean) recovery rate (as a percentage of the initial claim) is 5.8% (27.4%) for Chapter 7 while it is 79.2% (69.4%) for Chapter 11. Further, the fraction of firms which have 0% recovery is 79% for Chapter 7 and 0% for Chapter 11. These means are similar to those in by Acharya, et. al. [3] who document (Table 8) that the mean recovery rate for Chapter 7 is 26.38% and for Chapter 11 is 68.43%.¹

As Bris, et. al. [8] point out, whether a corporation files for Chapter 7 or Chapter 11 is endogenous and self-selection can contaminate the estimation of bankruptcy costs. In particular, if firms self-select, then it could be misleading to compare the cost of procedures, without controlling for endogeneity of chapter choice. The authors carefully attempt to control for the self-selection into bankruptcy chapter (Chapter 7 or Chapter 11) in their regressions.

¹These (PE) numbers are themselves averages between no industry distress and industry distress states.

To understand the determinants of corporate bankruptcy choices and complement empirical studies like Bris, et. al. [8], we extend the basic structural corporate finance models of Cooley and Quadrini [11], Gomes [17], and Hennessy and Whited [19] to incorporate a nontrivial bankruptcy choice.² We compare firm dynamics and productivity between a model where both bankruptcy options are available to counterfactuals without both options to evaluate possible inefficiencies of such policies.³ Adding a nontrivial bankruptcy choice to an environment where cash flows can turn negative (due to fixed costs as in Hopenhayn [20]) has important implications beyond the selection issues raised above. For instance, it implies that liquidation arises in equilibrium for a subset of firms in our model, while it does not in Cooley and Quadrini [11] nor Hennessy and Whited [19]. It even shows up methodologically since with liquidation costs which depend on the amount of collateral, here we must expand the state space and cannot simply use net worth. Further, these papers only consider take-it-or-leave-it bargaining in renegotiation.⁴ Interestingly, we find that a change in bankruptcy laws along the lines of that proposed by Aghion, Hart, and Moore [4] and currently considered by the American Bankruptcy Institute can have a significant impact on interest rates, equilibrium capital structure, the firm size distribution, and welfare.⁵

2 Bankruptcy Facts from Compustat

Given the fact that the vast majority of empirical corporate finance papers use data from Compustat, we organize bankruptcy facts using Compustat data from 1980-2012. This is obviously a different sample than that in Bris, et. al. [8]. Some of our facts are similar to those in Bris, et. al. [8] (e.g. the fraction of Chapter 11 bankruptcies relative to the total number of bankruptcies) while other facts differ (firms are more highly levered in their sample). We note, however, that there can be substantial differences in reported bankruptcy facts across datasets. For instance, bankruptcy statistics on *all* business filings from the U.S. Courts (<http://www.uscourts.gov/Statistics/BankruptcyStatistics.aspx>) suggest that the Bris, et. al. [8] sample as well as ours overstates the proportion of Chapter 11 business bankruptcies. For instance, in the U.S. Courts dataset (which includes smaller firms), the fraction of Chapter 11 business bankruptcies to total business bankruptcies was roughly 25% for the year ending in December 2013.

²Other closely related papers include Arellano, Bai and Zhang [5], D’Erasmus and Moscoso Boedo [14], Khan, Senga, and Thomas [22], Meh and Terajima [23] and Cooper and Ejarque [12].

³In an important corporate finance paper, Brodie, et. al. [9] study Ch. 7 vs. Ch. 11 decision problem but in a much simpler model with exogenous cash flows and initial bond finance of fixed investment.

⁴Eraslan [16] studies Chapter 11 in a more general bargaining environment

⁵The idea that policies which affect the cost to exit can have important implications for entry, the firm size distribution, and welfare is not new. For instance, Hopenhayn and Rogerson [21] (see Table 3) find that firing costs can have a significant impact on hiring, the firm size distribution, and welfare.

Besides simply comparing characteristics of firms in the state of bankruptcy as in [8] or the U.S. Courts, here we also compare characteristics of firms that are not bankrupt to those which are bankrupt. Table 1 displays a summary of some key differences between Chapter 7, Chapter 11 and Non-Bankrupt firm variables which have analogues in our model (see Appendix A1 for a detailed description of the data). Since there can be substantial differences between the median and mean of these variables, the table provides both. In Figures 1 and 2 we graph the conditional distributions of some of the key variables in the model. Further, we test whether the means differ between Chapter 7, Chapter 11, and Non-bankrupt.

Table 1: Balance Sheet and Corporate Bankruptcies 1980-2012

Moment						
Frequency of Exit (%)	0.71					
Fraction of Exit by Ch 7 (%)	59.23					
Frequency of (all) Bankruptcy (%)	0.87					
Fraction of Chapter 11 Bankruptcy (%)	80.82					
	Non-Bankrupt		Chapter 11		Chapter 7	
	Avg.	Median	Avg.	Median	Avg.	Median
Capital (millions 1983\$)	932.72	34.59	438.34*,***	75.92	79.32**	20.65
Cash (millions 1983\$)	120.78	9.35	58.16*,***	5.25	11.72**	2.99
Assets (millions 1983\$)	1053.49	55.25	496.61*,***	87.21	91.04**	27.37
Op. Income (EBITDA) / Assets (%)	11.19	15.84	-13.18*	-2.31	-17.48**	-4.97
Net Debt / Assets (%)	25.87	15.79	61.57*,***	37.19	48.89**	37.58
Total Debt / Assets (%)	57.43	35.57	83.07*,***	53.49	77.32**	55.93
Frac. Firms with Negative Net Debt (%)	26.26	-	17.49*	-	18.37**	-
Secured Debt / Total Debt (%)	43.94	40.90	47.34*	43.96	49.42**	48.02
Interest Coverage (EBITDA/Interest)	13.73	4.84	-0.09*	-0.15	-6.18**	-0.31
Equity Issuance / Assets (%)	8.03	0.09	4.90*	0.00	5.08**	0.00
Fraction Firms Issuing Equity (%)	27.62	-	35.22*	-	14.11**	-
Net Investment / Assets (%)	1.36	0.58	-4.23*	-4.44	-3.95**	-3.44
Dividend / Assets (%)	5.98	3.12	3.24*,***	1.29	4.17	1.64
Z-score	3.74	3.19	-1.22*,***	0.11	-1.42**	0.37
DD Prob. of Default (%)	2.16	0.00	3.67*	1.22	4.55**	1.76

Note: See appendix A1 for a detailed definition of variables and the construction of bankruptcy and exit indicators. Medians (Average) reported in the table correspond to the time series average of the cross-sectional median (mean) obtained for every year in our sample. Test for differences in means at 10% level of significance: * denotes Chapter 11 different from Non-bankrupt, ** denotes Chapter 7 different from Non-bankrupt, *** denotes Chapter 11 different from Chapter 7.

We follow the classification into Chapter 7 and Chapter 11 bankruptcy used by Duffie, Saita and Wang [15]. Chapter 7 in Table 1 corresponds to values for the final observation of a firm that exits via a Chapter 7 bankruptcy. Chapter 11 refers to an observation in the initial period

of a Chapter 11 bankruptcy. Non-bankrupt identifies annual observations of firms that are not in the state of bankruptcy (i.e. firms which never declare bankruptcy as well as observations of firms before they declare bankruptcy excluding the above). To be consistent with the way that the U.S. Bureau of the Census constructs its exit statistics, a deleted firm (i.e. a firm that disappears from our sample) is counted as a firm that exits if its deletion code is not 01 (M & A), 02 (Bankruptcy which we associate with Ch.11), 04 (Reverse Acquisition), 09 (going private) or 07 and 10 (other). For example, this means that firms which are acquired or go from public to private are not counted as exiting. Code 03 is defined as liquidation, which we associate with Chapter 7. The only firm assets in the model we present below are physical capital and cash, so our measure of assets tends to be lower than other studies which implies ratios like leverage tend to be higher than other studies. In the Appendix, we provide more information about the frequencies of those events.

Table 1 documents that exit rates (fraction of deletions to all firms in a given year) are small (0.7%) in our sample and 60% of exits are by Chapter 7 liquidation. The fraction of all firms declaring bankruptcy is also small (0.87%) in our sample and 81% of bankruptcies are by Chapter 11 (as in [8]).

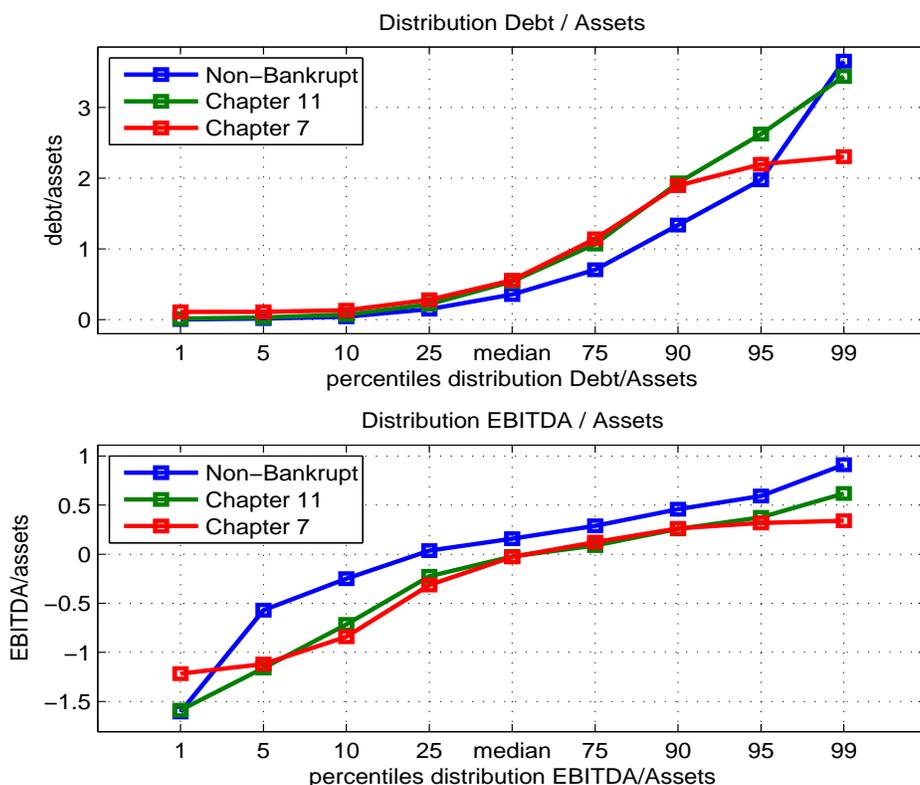
Since firms in our model choose physical capital and net debt (total debt minus cash), we examine differences in size measured by total assets (capital plus cash) as well as its composition. Non-bankrupt firms are bigger than Chapter 11 firms which in turn are bigger than Chapter 7 firms. In all cases, the differences in mean are statistically significant (at the 10% level).

Earnings before interest, taxes, depreciation, and amortization (EBITDA) measures a firm's profitability. Negative values generally indicate a firm has fundamental profitability issues, while a positive value does not necessarily mean it is profitable since it generally ignores changes in working capital as well as the other terms described above. The median and mean ratio of EBITDA to assets is negative for both Chapter 11 and Chapter 7 firms, while it is positive for Non-bankrupt firms. Differences in mean between Non-bankrupt versus Chapter 11 and Chapter 7 are statistically significant, but not statistically significant between Chapter 11 versus 7. These statistics accord well with the idea that bankrupt firms have profitability problems.

We provide several measures of leverage. Net debt is measured as debt minus cash, where negative values imply that the firm is highly liquid. We find that both median and mean net debt or total debt to assets are highest for Chapter 11 and lowest for Non-bankrupt firms. Statistical significance of differences in mean leverage exist across all types. The time average of the fraction of firms with negative net debt (i.e. liquid firms) is higher for Non-bankrupt than bankrupt firms. There is a statistically significant difference in means between bankrupt and Non-bankrupt, but not between Chapter 11 and 7. The ratio of secured to total debt is

highest for Chapter 7 and lowest for nonbankrupt firms. There is a statistically significant difference in means between Non-bankrupt versus Chapter 11 and Chapter 7, but not between Chapter 11 versus Chapter 7. Interest coverage is measured as the ratio of EBITDA to interest expenses. It is generally thought that a ratio less than one is not sustainable for long. Here we see that both mean and median interest coverage is positive and large for nonbankrupt firms while it is in general negative for bankrupt firms. There are insignificant statistical differences in mean between the two bankruptcy choices, but the differences are statistically significant between bankrupt and Non-bankrupt.

Figure 1: Distribution of Debt/Assets and EBITDA/Assets

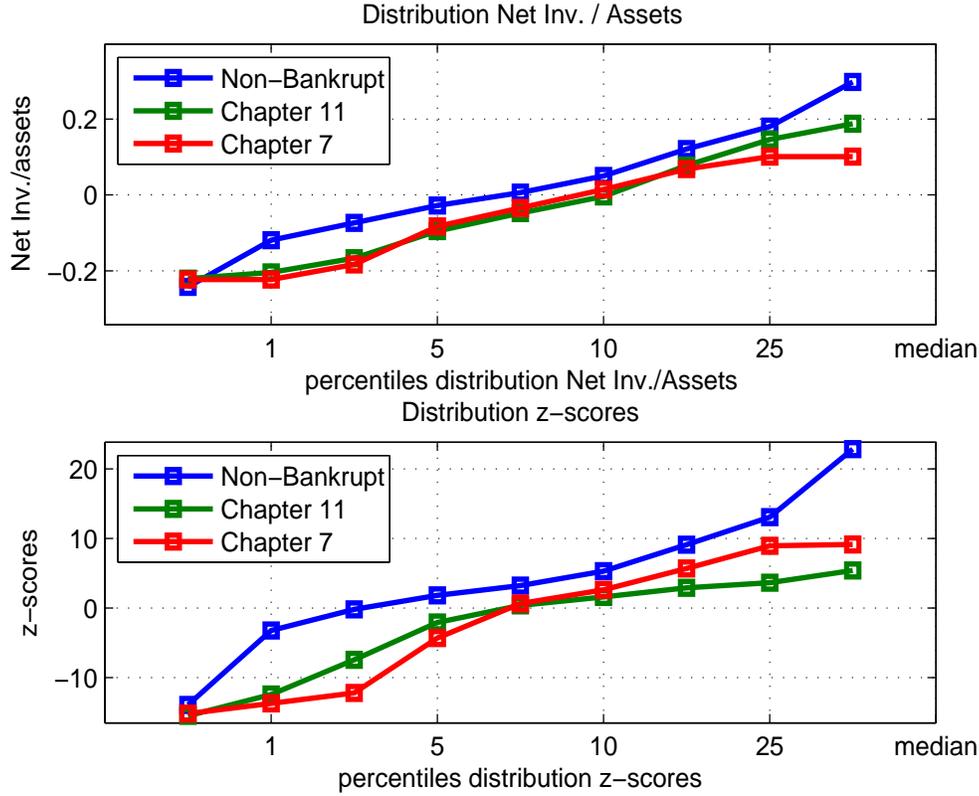


Equity issuance is highest for Nonbankrupt firms and statistically significant relative to bankrupt firms, but statistically insignificant between bankrupt choices. The time average of the fraction of firms issuing equity in any given period is highest for Chapter 11 and lowest for Chapter 7, though the differences are only statistically significant between Nonbankrupt and bankrupt.

Median and average net investment (gross investment minus depreciation) is positive for Nonbankrupt firms and negative for bankrupt firms. The differences between Non-bankrupt versus bankrupt are statistically significant, but not statistically significant between Chapter

11 and Chapter 7. Dividend payouts are highest for nonbankrupt firms and lowest for Chapter 11 firms. In terms of means, there is a statistically significant difference between Chapter 11 and other types of firms.

Figure 2: Distribution of Net. Investment/Assets and z-scores



We also consider two well accepted measures of corporate default probabilities from the finance literature: “z-scores” and “distance-to-default (DD)”. The Altman [1] z-score is a linear combination of five common firm level ratios: working capital to assets, retained earnings to assets, earnings before interest and taxes, market value of equity to book value of total liabilities, and sales to total assets. While simplistic, Altman’s z-score is widely used by practitioner’s as a predictor of default within the next two years with values greater than 2.9 safe while values less than 1 indicative of distress. Table 1 documents that both the median and average z-score for nonbankrupt firms exceeds 3, while z-scores for both Chapter 7 and 11 are generally below 1. All differences in mean are statistically significant. The DD measure is based upon an estimate of the asset value and volatility of a firm using an option pricing model, along with the observed book value of debt and market value of equity. To compute estimates of asset value and volatility we use an iterative procedure as in Duffie, et. al. [15] (see Appendix for a full description of the construction of DD). Table 1 documents that the

average DD is significantly higher for firms we classify as bankrupt than nonbankrupt.

In summary, nonbankrupt firms: (i) are bigger than bankrupt firms; (ii) are profitable while bankrupt firms are not; (iii) have lower leverage than bankrupt firms; (iv) have lower interest expenses relative to their cash flow; (v) do not have statistically significant differences in equity issuance; (vi) have positive net investment as opposed to negative net investment for bankrupt firms; (vii) have higher dividend payouts than bankrupt firms; and (viii) have lower likelihoods of default as measured by practitioner’s “models” of default. Further, in terms of statistical significance, there is resounding support for differences between bankrupt and Non-bankrupt firms but slightly less so between firms which choose Chapter 11 versus Chapter 7. This latter result could be due to the small sample size of bankrupt firms.

3 Environment

We consider a discrete time, general equilibrium model where heterogeneous firms produce a homogeneous good and issue short term defaultable debt and costly equity to undertake investment and dividend choices. Since firms can choose chapter 7 or chapter 13 bankruptcy, competitive lenders must attempt to predict default decisions of the firms they are lending to when determining the price of debt. There is a representative household which maximizes lifetime utility and whose income comes from wages and dividends on the shares that the representative household holds in every firm. We will focus our attention on a stationary equilibrium characterized by a measure of firms endogenously distributed across capital and net debt.

3.1 Firms and Technology

Competitive firms produce a homogeneous good that can be consumed by households or can be used as capital. Firm j maximizes the expected discounted value of dividends

$$E_0 \sum_{t=0}^{\infty} (1+r)^{-t} d_{jt} \tag{1}$$

where d_{jt} denotes dividends in period t and $(1+r)^{-1}$ is the discount rate of the firm.⁶ Firms have access to a decreasing returns to scale production technology

$$y_{jt} = z_{jt} \left(k_{jt}^{\alpha} n_{jt}^{1-\alpha} \right)^{\nu}, \quad \alpha \in (0, 1), \nu \in (0, 1) \tag{2}$$

⁶Since there are no aggregate shocks in this model, to conserve on notation here we define the objective using a constant discount rate, which is consistent in equilibrium.

where $z_{jt} \in Z \equiv \{z^1, \dots, z^n\}$, is an idiosyncratic productivity shock, i.i.d. across firms, that follows a first order Markov Process with transition matrix $G(z_{jt+1}|z_{jt})$, $n_{jt} \in \mathbb{R}_+$ is labor input, and $k_{jt} \in K \subset \mathbb{R}_+$ is capital input. There is a fixed cost of production c_f , measured in units of output. Firms must pay this fixed cost in order to produce. Active firms own their capital and decide the optimal level of gross investment $i_{jt}^g = k_{jt+1} - (1 - \delta)k_{jt} = i_{jt}^n + \delta k_{jt}$ where i_{jt}^n is net investment. Firms pay capital adjustment costs

$$\Psi(k_{jt+1}, k_{jt}) \equiv \frac{\psi}{2} \left(\frac{i_{jt}^g}{k_{jt}} \right)^2 k_{jt}. \quad (3)$$

In any given period, firm j 's operating income (EBITDA) is given by

$$\pi_{jt} = y_{jt} - w_t n_{jt} - c_f \quad (4)$$

where w_t is the competitively determined real wage. Inputs can be financed from three sources: (i) one period non-contingent debt $b_{jt+1} \in B \subset \mathbb{R}$ at discounted price q_{jt} ; (ii) current cash flow and internal savings; and (iii) external equity at cost λ to raise funds (so with our assumptions on taxes, firms will never find it optimal to simultaneously pay dividends and issue equity). Taxable income is $\Upsilon_{jt} = \pi_{jt} - \delta k_{jt} - \left(\frac{1}{q_{jt}} - 1 \right) \frac{b_{jt+1}}{(1+r)}$ (i.e. operating profits less economic depreciation less discounted interest expense) and corporate taxes are

$$T_{jt}^c = \mathbf{1}_{\{\Upsilon_{jt} \geq 0\}} \tau_c \cdot \Upsilon_{jt} \quad (5)$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function that takes value one if the condition in brackets holds and zero otherwise.⁷

The after-tax net cash flow to equity holders is given by

$$d_{jt} = \begin{cases} (1 - \tau_d) e_{jt} & \text{if } e_{jt} \geq 0 \\ e_{jt} - \lambda(e_{jt}) & \text{if } e_{jt} < 0 \end{cases} \quad (6)$$

where

$$e_{jt} = \pi_{jt} - T_{jt}^c - i_{jt}^g - b_{jt} + q_{jt} b_{jt+1} - \Psi(k_{jt+1}, k_{jt}). \quad (7)$$

In particular, firms pay dividends if $e_{jt} \geq 0$ which incurs dividend taxes τ_d . If $e_{jt} < 0$, firms pay external finance costs $\lambda(e_{jt})$. Note that provided taxable income is positive, the tax benefit of a unit of debt is given by $(1 - \tau_d) \cdot \tau_c \left(\frac{1}{q_{jt}} - 1 \right) / (1 + r) > 0$.

Firms can enter by paying a cost κ . After paying this cost, which is financed by either equity

⁷As in Strebulaev and Whited [26] we assume the firm takes the present value of the interest tax deduction in the period in which it issues debt. This allows us to avoid adding another state variable. Further, for simplicity, unlike Hennessy and Whited [19] we assume there are no loss limitations (i.e. $\tau_c^+ = \tau_c^- = \tau_c$).

or debt issue, firms observe their initial level of productivity z_{j0} drawn from the stationary distribution $\bar{G}(z)$ derived from $G(z_{jt+1}|z_{jt})$. We denote the mass of new entrants as M .

3.2 Financial Markets

Firms finance operations either through debt or equity. Equity issuance costs are an increasing function $\lambda(e_{jt})$ of the amount of equity issued and we normalize the number of shares per firm to 1. A share is a divisible claim on the dividends of the firm.

Competitive lenders have access to one period risk free discount bonds at after-tax price q_t^B . Loans mature each period and their price q_{jt} depends on how much the firm borrows b_{jt+1} as well as other characteristics like firm capital holdings k_{jt+1} (since this affects liquidation value) and current productivity z_{jt} . Debt is non-contingent in the sense that it does not depend on future z_{jt+1} .⁸

Firms can default on their debt triggering a bankruptcy procedure. To resemble U.S. law, we allow for two default options:

1. **Chapter 7 liquidation:** Firm j liquidates its assets at firesale discount $s_7 < 1$ which it uses to pay debts, incurs a bankruptcy cost c_7 , and exits. Shareholders obtain (pre-tax): $\max\{s_7 k_{jt} - b_{jt} - c_7, 0\}$. Lenders obtain $\min\{b_{jt}, \max\{s_7 k_{jt} - c_7, 0\}\}$.
2. **Chapter 11 reorganization:** Firm j and lenders renegotiate the defaulted debt, bargain over the repayment fraction ϕ_{jt} (where the firm's bargaining weight is given by θ), the firm pays bankruptcy cost c_{11} , reduces its debt to $\phi_{jt} b_{jt}$ (where $\phi_{jt} \in [0, 1]$), faces equity finance costs $\lambda_{11}(e_{jt})$, debt finance costs $\lambda_{11}^b \leq 1$, and continues operating (i.e. does not exit).

When making a loan to a firm, lenders take into account that in the case of default they can recover up to a fraction of the original loan. As described above, the recovery rate of a loan depends on the bankruptcy procedure chosen by the firm. In the case of a Chapter 7 liquidation, when making a loan of size b_{jt} in period t , lenders can expect to recover in period $t + 1$, $\min\{b_{jt+1}, \max\{s_7 k_{jt+1} - c_7, 0\}\}$ where s is the scrap price of the firm's capital (which serves as collateral).⁹ If the firm chooses to reorganize (i.e., Chapter 11), the recovery rate in period $t + 1$ will be ϕ_{jt+1} . That is, lenders will recover this fraction of debt that they agree upon during the reorganization process. We assume the negotiation over recovery rate solves a Nash Bargaining problem where the firm's weight is θ and the lender's weight is $1 - \theta$.

Of course, a firm can choose to exit without defaulting at any point in time. In this case, the firm liquidates its assets (we assume at par) and pays its debts in full.

⁸Our use of equilibrium price menus which depend on agent characteristics in the presence of default in a quantitative model is similar to that in Chatterjee, et. al. [10] and Hennessy and Whited [19].

⁹Hennessy and Whited [18] make a similar assumption. Stromberg [27] finds that asset fire sales and resales to management can lead to low salvage values and striking inefficiencies in the Chapter 7 procedure.

3.3 Households

In any period t , households choose a stream of consumption C_t , shares $\{S_{jt+1}\}_j$ of incumbent and entrant firms, and risk free bonds B_{t+1} to maximize the expected present discounted value of utility given by

$$\max E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t) \right] \quad (8)$$

subject to

$$C_t + \int p_{jt} S_{jt+1} dj + q_t^B B_{t+1} = w_t + \int (p_{jt} + d_{jt}) S_{jt} dj + B_t + T_t^h \quad (9)$$

where p_{jt} is the after dividend stock price of firm j , q_t^B is the after-tax price of the risk free discount bond, and T_t^h are lump sum taxes/transfers for households. The marginal income tax τ_i applied to wage and interest earnings is rebated back to households in T_t^h . It should be understood that the stock price of a firm which exits is taken to be zero and that since preferences do not include leisure, households supply their unit of labor inelastically.

3.4 Timing

In this section, we describe the timing of the model. At the beginning of period t :

1. Productivity z_{jt} is realized. The state space for incumbent firm j is given by $\{z_{jt}, k_{jt}, b_{jt}\}$.
2. Bankruptcy decision for incumbent firms.
 - If the firm chooses to declare bankruptcy, it chooses whether to exit by Chapter 7 liquidation or to continue via Chapter 11 reorganization.
 - If the firm chooses Chapter 7, it incurs costs (c_7, s_7) , pays “final dividends” $d_{jt} = (1 - \tau_i) \max\{s_7 k_{jt} - b_{jt} - c_7, 0\}$, and exits.¹⁰
 - If the firm chooses Chapter 11, it bargains with lenders over a recovery rate ϕ_{jt} and incurs costs c_{11} . Once the firm and lenders agree on a recovery rate, the firm repays $\phi_{jt} b_{jt}$ and continues operating. It chooses an amount of capital k_{jt+1} and finances it via internal funds, new debt b_{jt+1} at price $\lambda_{11}^b q(k_{jt+1}, b_{jt+1}, z_{jt})$ and/or equity issuance with costs $\lambda_{11}(e_{jt})$.
 - If the firm chooses not to declare bankruptcy, it repays in full and chooses whether to continue or to exit.
 - If firm j chooses to continue after repaying b_{jt} , it chooses the amount of capital k_{jt+1} and finances it via internal funds, debt b_{jt+1} at price $q(k_{jt+1}, b_{jt+1}, z_{jt})$

¹⁰Note that because of our timing assumptions, the taxation issues about applying net operating losses in Chapter 7 are absent.

and/or equity issuance at cost λ .

- If firm j chooses to exit after repaying b_{jt} , it pays “final dividends” $d_{jt} = (1 - \tau_i)(k_{jt} - b_{jt})$ if $k_{jt} \geq b_{jt}$ and $d_{jt} = (1 + \lambda)(k_{jt} - b_{jt})$ otherwise. Note that repayment avoids both bankruptcy costs c_7 as well as salvage value s .
- 3. Entry decision. Potential entrants decide whether to start a firm or not. If they enter, they pay the entry cost κ and choose their initial level of capital by issuing equity at cost $\lambda_E(e_{jt})$ or debt. The initial productivity shock is drawn from $\overline{G}(z)$.
- 4. Households choose shares and bonds, which given earnings and taxes determines their consumption. If the household chooses to purchase the stock of an entrant, then $S_{jt+1} = S_{jt}$ and $d_{jt} = -k_{jt+1} + q(k_{jt+1}, b_{jt+1})b_{jt+1} - \kappa - \lambda_E(-k_{jt+1} + q(k_{jt+1}, b_{jt+1})b_{jt+1} - \kappa)$.

4 Equilibrium

We consider only stationary equilibria of the model. In what follows we use the notation that $x_t = x$ and $x_{t+1} = x'$. Rather than refer to a given firm by its name j , it will be named by its place in the cross-sectional distribution of firms $\Gamma(z, k, b)$. To save on notation, we avoid making the dependence of decision rules on prices explicit.

4.1 Recursive Representation of the Firm’s Problem

An incumbent firm starts the period with capital k , debt b and productivity z . First, it makes its bankruptcy decision. The value of the firm $V(z, k, b)$ is defined as follows:

$$V(z, k, b) = \max_{\Delta \in \{0,1\}} V_{\Delta}(z, k, b) \quad (10)$$

where $\Delta = 0$ denotes the decision not to declare bankruptcy while $\Delta = 1$ corresponds to the decision to declare bankruptcy.

Conditional on the bankruptcy choice, the firm must choose whether it wants to exit $x = 1$ or not $x = 0$. In particular,

$$V_{\Delta}(z, k, b) = \max_{x \in \{0,1\}} V_{\Delta}^x(z, k, b) \quad (11)$$

Note that a firm which chooses not to declare bankruptcy ($\Delta = 0$) still must choose whether it wants to exit $x = 1$ or not $x = 0$. On the other hand, a decision to declare bankruptcy and choose exit (i.e. $(\Delta = 1, x = 1)$) implies a Chapter 7 liquidation while a decision to declare bankruptcy and not exit (i.e. $(\Delta = 1, x = 0)$) implies a Chapter 11 reorganization.

If the firm chooses not to declare bankruptcy and not to exit (i.e. $(\Delta = 0, x = 0)$), then:

$$V_0^0(z, k, b) = \max_{n \geq 0, k' \geq 0, b'} \left\{ d + (1+r)^{-1} E_{z'|z} [V(z', k', b')] \right\} \quad (12)$$

s.t.

$$e = \pi - T^c(k, z, k', b') - (k' - (1 - \delta)k) - b + q(b', k', z)b' - \Psi(k', k)$$

$$d = \begin{cases} (1 - \tau_d)e & \text{if } e \geq 0 \\ e - \lambda(e) & \text{if } e < 0 \end{cases}$$

We denote the optimal labor, capital, debt, and dividend decision rules by $n = h_0^n(z, k, b)$, $k' = h_0^k(z, k, b)$, $b' = h_0^b(z, k, b)$, and $d = h_0^d(z, k, b)$, respectively.

If the firm chooses not to declare bankruptcy and to exit (i.e. $(\Delta = 0, x = 1)$), the dividend policy implies

$$V_0^1(z, k, b) = \begin{cases} (1 - \tau_i)(k - b) & \text{if } k \geq b \\ k - b - \lambda(k - b) & \text{if } k < b \end{cases}. \quad (13)$$

That is, if the firm wishes to exit without using the limited liability benefit that Chapter 7 provides in the event of $k < b$, it must pay off its debt by an equity infusion.¹¹

If the firm chooses to declare bankruptcy but not to exit (i.e. a Chapter 11 reorganization $(\Delta = 1, x = 0)$), we can define payoffs taking as given the recovery rate $\phi(z, k, b)$ which will be determined by Nash bargaining in subsection 4.4 below as:

$$V_1^0(z, k, b) = \max_{n \geq 0, k' \geq 0, b'} \left\{ d + (1+r)^{-1} E_{z'|z} [V(z', k', b')] \right\} \quad (14)$$

s.t.

$$e = \pi - T^c(k, z, k', b') - (k' - (1 - \delta)k) - \phi(z, k, b)b + \lambda_{11}^b q(k', b', z)b' - \Psi(k', k) - c_{11}$$

$$d = \begin{cases} (1 - \tau_d)e & \text{if } e \geq 0 \\ e - \lambda_{11}(e) & \text{if } e < 0 \end{cases}$$

We allow the external finance costs λ_{11}^b and $\lambda_{11}(e)$ to differ for a firm under reorganization. We denote the optimal labor, capital, debt, and dividend decision rules by $n = h_1^n(z, k, b)$, $k' = h_1^k(z, k, b)$, $b' = h_1^b(z, k, b)$, and $d = h_1^d(z, k, b)$ respectively.

Finally, if the firm chooses to declare bankruptcy and to exit (i.e. a Chapter 7 liquidation $(\Delta = 1, x = 1)$),

$$V_1^1(z, k, b) = (1 - \tau_i) \max\{s_7 k - b - c_7, 0\}. \quad (15)$$

¹¹Obviously, this won't happen in equilibrium but the option must be allowed for completeness.

4.2 Entrants

In order to draw an initial productivity z_0 , entrants must pay κ . New firms are created with an initial value of equity raised by issuing new shares and debt. The value of a potential entrant is given by:

$$V_E = \max_{k' \geq 0, b'} \left\{ d_E + (1+r)^{-1} \sum_{z'} V(z', k', b') \bar{G}(z') \right\} \quad (16)$$

where

$$d_E = -k'_E + q(k'_E, b'_E)b'_E - \kappa - \lambda_E(-k'_E + q(k'_E, b'_E)b'_E - \kappa). \quad (17)$$

We denote the optimal capital and borrowing decision rules by k'_E and b'_E .

4.3 Lender's Problem

Lenders pool risky corporate loans and borrow from households in the risk free market at price q^B . The profit on a loan of size b' has two important components. First, the probability of default $\Lambda(b', k', z)$ is given by:

$$\Lambda(b', k', z) = \sum_{\{z' \in D_7(k', b')\} \cup \{z' \in D_{11}(k', b')\}} G(z'|z), \quad (18)$$

where $D_7(k, b)$ and $D_{11}(k, b)$ denote the Chapter 7 and Chapter 11 default sets respectively defined as:

$$\begin{aligned} D_7(k, b) &= \{z \in Z : \Delta(z, k, b) = 1 \ \& \ x(z, k, b) = 1\}, \\ D_{11}(k, b) &= \{z \in Z : \Delta(z, k, b) = 1 \ \& \ x(z, k, b) = 0\}. \end{aligned}$$

The second important component of a lender's profit is the expected recovery rate. If the firm chooses to file for Chapter 7 bankruptcy the lender recovers $\min\{b', \max\{s_7 k' - c_7, 0\}\}$. If the firm chooses to reorganize under Chapter 11 the lender will recover $\phi(z', k', b')b'$ which is the solution to a bargaining game between the firm and the lender. Thus, we can write the lender's profit function as follows:

$$\begin{aligned} \Omega(b', k', z) &= -q(b', k', z)b' + q^B[1 - \Lambda(b', k', z)]b' \\ &\quad + q^B \sum_{z' \in D_7(k', b')} \min\{b', \max\{s_7 k' - c_7, 0\}\} G(z'|z) \\ &\quad + q^B \sum_{z' \in D_{11}(k', b')} \phi(z', k', b')b' G(z'|z). \end{aligned} \quad (19)$$

4.4 Reorganization

Upon reaching a bargaining agreement, the value of defaulted debt is reduced to a fraction φ of the unpaid debt b . The value of an agreement of size φ to the firm is:

$$V^R(z, k, b; \varphi) = \max_{n, b', k'} \left\{ d + (1+r)^{-1} E_{z'|z} [V(k', b', z')] \right\}$$

s.t.

$$e = \pi - T^c(k, z, k', b') - (k' - (1 - \delta)k) - \varphi b + \lambda_{11}^b q(k', b', z) b' - \Psi(k', k) - c_{11}$$

$$d = \begin{cases} (1 - \tau_d)e & \text{if } e \geq 0 \\ e - \lambda_{11}(e) & \text{if } e < 0 \end{cases}$$

After the repayment of a fraction φ of debt, the firm chooses the optimal level of investment, can issue debt or equity which may cost a different amount during renegotiation, and continues operating.

Since either the borrower or lender in the renegotiation phase of Chapter 11 have a right to declare Chapter 7 bankruptcy, we assume that the threat points are equal to the payoffs associated with Chapter 7 liquidation.¹² We allow there to be differences in the salvage value of Chapter 11 firms which end up being liquidated (i.e. (s_{11})). In that case, the surplus for the firm is

$$W^R(z, k, b; \varphi) = V^R(z, k, b; \varphi) - (1 - \tau_i) \max\{s_{11}k - b - c_{11}, 0\}. \quad (20)$$

Since the value of an agreement for the lender is φb (i.e., the recovery on defaulted debt), the surplus for the lender is:

$$W^L(z, k, b; \varphi) = \varphi b - \min\{b, \max\{s_{11}k - c_{11}, 0\}\}. \quad (21)$$

The recovery rate is then the solution to the following Nash Bargaining problem:

$$\phi(z, k, b) \equiv \arg \max_{\varphi \in [0, 1]} [W^R(z, k, b; \varphi)]^\theta [W^L(z, k, b; \varphi)]^{1-\theta} \quad (22)$$

s.t.

$$\begin{aligned} W^R(z, k, b; \varphi) &\geq 0, \\ W^L(z, k, b; \varphi) &\geq 0. \end{aligned}$$

Due to the general equilibrium nature of our problem it is difficult to sign the effect of

¹²As stated on p. 663 in Eraslan [16], “If no progress (in Chapter 11) is made toward agreement, then the court can decide to convert the case to Chapter 7.” See also “Conversion or Dismissal” in <http://www.uscourts.gov/FederalCourts/Bankruptcy/BankruptcyBasics/Chapter11.aspx>

changes of firm bargaining power (θ) on the fraction it repays lenders ϕ . In all the equilibria we present, however, a rise in bargaining power leads to lower repayment. Notice further that if $\theta = 1$, then the lender's surplus in (21) will be zero. In that case, an equilibrium with positive debt where $s_{11}k - c_11 < 0$ implies $\phi = 0$ (i.e. if a firm with little capital has all the bargaining power, it doesn't repay debt in reorganization). However, if $s_{11}k - c_11 \geq 0$, then even with $\theta = 1$, creditors will receive some repayment (i.e. $\phi > 0$). Thus, if high capital firms with debt enter reorganization (something which happens in the data and under our parameterization), then even if creditors have no bargaining it is possible that there will be some payment in Chapter 11.

4.5 Household's problem

The first order conditions for the household's problem (8)-(9) are given by:

$$\begin{aligned} B_{t+1} &: q_t^B U'(C_t) = \beta E_t [U'(C_{t+1})] \\ S_{jt+1}, \forall j &: p_{jt} U'(C_t) = \beta E_t [U'(C_{t+1}) (p_{jt+1} + d_{jt+1})] \end{aligned}$$

In a steady state this implies:

$$q_t^B = \beta \tag{23}$$

$$p_{jt} = \beta E_t [p_{jt+1} + d_{jt+1}] \tag{24}$$

To characterize stock prices, consider the case of an incumbent firm and let $p(z, k, b) = V(z, k, b) - d(z, k, b)$. Then it is straightforward to show that (24) is equivalent to (10) or

$$\begin{aligned} p(z, k, b) &= \beta E_{z'|z} [p(z', k', b') + d(z', k', b')] \\ \iff V(z, k, b) - d(z, k, b) &= (1 + r)^{-1} E_{z'|z} [V(z', k', b')] \end{aligned} \tag{25}$$

In the case of purchasing a stock of an entrant, $S_E = S' = S$, in which case $p_j S_{jt+1}$ and $p_j S_{jt}$ cancel and the initial equity injection given by d_E in (17) is accounted for in the household's budget set (9).

An implication of (25) is that firm optimization in a steady state implies

$$(1 + r)^{-1} = \beta. \tag{26}$$

4.6 Cross-Sectional Distribution

Let $\bar{K} \subset K$, $\bar{B} \subset B$ and $\bar{Z} \subset Z$. Further, let $(\Delta(z, k, b), x(z, k, b)) = \arg \max_{\Delta \in \{0,1\}, x \in \{0,1\}} V_{\Delta}^x(z, k, b)$. The law of motion for the cross-sectional distribution of firms is given by:

$$\begin{aligned} \Gamma'(\bar{K}, \bar{B}, \bar{Z}; M, w) &= \int_{\bar{K}, \bar{B}} \sum_{\bar{Z}} \left\{ \int_{K, B} \sum_Z (1 - x(z, k, b)) \left[\mathbf{1}_{\{\Delta(k, b, z)=0\}} \mathbf{1}_{\{k'=h_0^k(z, k, b), b'=h_0^b(z, k, b)\}} \right. \right. \\ &\quad \left. \left. + \mathbf{1}_{\{\Delta(k, b, z)=1\}} \mathbf{1}_{\{k'=h_1^k(z, k, b), b'=h_1^b(z, k, b)\}} \right] G(z'|z) \Gamma(dk, db, z) \right\} dk' db' \\ &\quad + M \sum_{\bar{Z}} \mathbf{1}_{\{k'_E, b'_E\}} \bar{G}(z) \end{aligned} \quad (27)$$

where M is the mass of new entrants.

4.7 Definition of Equilibrium

A stationary Markov Equilibrium is a list $\{V^*, w^*, r^*, q^{B*}, q^*, \phi^*, p^*, D_7^*, D_{11}^*, \Lambda^*, \Gamma^*, M^*, C^*, B'^*, S'^*, T^*\}$ such that:

1. Given w, r, q, ϕ , the value function V^* is consistent with the firm's optimization problem in (12)-(15).
2. Given V, w, r, q , the recovery rate $\phi^*(k, b, z)$ solves the bargaining problem (22).
3. The probability of default Λ^* in (18) and the sets D_i^* for $i = 7, 11$ are consistent with firm decision rules.
4. The equilibrium loan price schedule is such that lenders earn zero profits in expected value on each contract. That is, at $q^*(b', k', z)$, $\Omega^*(b', k', z) = 0$ in (19).
5. The cost of creating a firm is such that $V_E^* = 0$ in (16).
6. $\Gamma^*(z, k, b)$ and M^* in (27) is a stationary measure of firms consistent with firm decision rules and the law of motion for the stochastic variables.
7. Given (w, q^B, p) and taxes/transfers T^h , households solve (8)-(9) and (q^{B*}, p^*, r^*) are consistent with (23), (24), and (26).
8. Labor, bond and stock markets clear at w^*, q^{B*}, p^* or:

$$\begin{aligned} \int_{K, B} \sum_Z (1 - x(z, k, b)) \left[\mathbf{1}_{\{\Delta(k, b, z)=0\}} h_0^n(z, k, b) + \mathbf{1}_{\{\Delta(k, b, z)=1\}} h_1^n(z, k, b) \right] \Gamma(dk, db, z) &= 1 \\ \int_{K, B} \sum_Z (1 - x(z, k, b)) \left[\mathbf{1}_{\{\Delta(k, b, z)=0\}} h_0^b(z, k, b) + \mathbf{1}_{\{\Delta(k, b, z)=1\}} h_1^b(z, k, b) \right] \Gamma(dk, db, z) &= B'^* \\ S'^* &= 1 \end{aligned}$$

9. Taxes/Transfers satisfy the government budget constraint:

$$T^h = T^d + T^B + T^7 + T^i + \mathbf{T}^c - T^L \quad (28)$$

where dividend taxes T^d are

$$T^d = \tau_d \int_{K \times B} \sum_z (1 - x(k, b, z)) \mathbf{1}_{\{e(k, b, z) \geq 0\}} e(k, b, z) \Gamma(dk, db, z),$$

taxes on interest earnings T^B are

$$T^B = \tau_i \tilde{q}^B \left(\frac{1}{\tilde{q}^B} - 1 \right) B',$$

at pre-tax bond price \tilde{q}^B , taxes to cover bankruptcy cost of liquidated firms T^7 are ¹³

$$T^7 = \int_{K \times B} \sum_z x(k, b, z) \mathbf{1}_{\{\Delta=1\}} c_7 \Gamma(dk, db, z),$$

income taxes on the final distribution by exiting firms T^i are

$$T^i = \tau^i \int_{K \times B} \sum_z x(k, b, z) \left\{ \mathbf{1}_{\{\Delta=0\}} (k - b) + \mathbf{1}_{\{\Delta=1\}} \max\{s_7 k - b - c_7, 0\} \right\} \Gamma(dk, db, z),$$

corporate taxes are

$$\begin{aligned} \mathbf{T}^c = \int_{K, B} \sum_Z (1 - x(z, k, b)) & \left[\mathbf{1}_{\{\Delta(k, b, z)=0\}} T^c(k, z, h_0^k(z, k, b), h_0^b(z, k, b)) \right. \\ & \left. + \mathbf{1}_{\{\Delta(k, b, z)=1\}} T^c(k, z, h_1^k(z, k, b), h_1^b(z, k, b)) \right] \Gamma(dk, db, z) \end{aligned}$$

and taxes necessary to cover ex-post losses associated with bankruptcy T^L are

$$T^L = q^B \int_{K \times B} \sum_z \{-\Lambda(z, k, b)b + \min\{b, \max\{s_7 k - c_7, 0\}\} + \phi(z, k, b)b\} \Gamma(dk, db, z).$$

Of course, by Walras' law the household budget constraint (9) implies the goods market clearing condition is satisfied and aggregate consumption is given by

$$C = Y - CF - I - \Lambda + X - BC - E \quad (29)$$

¹³Note that in general, bankruptcy costs for liquidated firms need to be recovered by taxes only for those firms that go bankrupt and have $sk - b - c_7 < 0$. However, since this condition always holds in equilibrium we omitted the corresponding indicator function.

where aggregate output is

$$Y = \int_{K,B} \sum_Z (1 - x(z, k, b)) \left[\mathbf{1}_{\{\Delta=0\}} z (k^\alpha (h_0^n)^{1-\alpha})^\nu + \mathbf{1}_{\{\Delta=1\}} z (k^\alpha (h_1^n)^{1-\alpha})^\nu \right] \Gamma(dk, db, z) \quad (30)$$

aggregate operating costs are

$$CF = \int_{K \times B} \sum_z (1 - x(k, b, z)) c_f \Gamma(dk, db, z), \quad (31)$$

aggregate investment plus investment adjustment costs are

$$I = \int_{K,B} \sum_Z (1 - x(z, k, b)) \left[\mathbf{1}_{\{\Delta=0\}} \left[h_0^k(z, k, b) - (1 - \delta)k + \Psi(h_0^k(z, k, b), k) \right] \right. \\ \left. + \mathbf{1}_{\{\Delta=1\}} \left[h_1^k(z, k, b) - (1 - \delta)k + \Psi(h_1^k(z, k, b), k) \right] \right] \Gamma(dk, db, z) \quad (32)$$

aggregate equity issuance costs are

$$\Lambda = \int_{K \times B} \sum_z (1 - x(k, b, z)) \mathbf{1}_{\{e(k,b,z) < 0\}} (\mathbf{1}_{\{\Delta=0\}} \lambda(e) + \mathbf{1}_{\{\Delta=1\}} \lambda_{11}^e(e)) \Gamma(dk, db, z), \quad (33)$$

final distributions from exiting firms are

$$X = \int_{K \times B} \sum_z x(k, b, z) \left\{ \mathbf{1}_{\{\Delta=0\}} (k - b) + \mathbf{1}_{\{\Delta=1\}} \max\{s_7 k - b - c_7, 0\} \right\} \Gamma(dk, db, z), \quad (34)$$

aggregate bankruptcy costs are

$$BC = \int_{K \times B} \sum_z \left\{ (1 - x(k, b, z)) \mathbf{1}_{\{\Delta=1\}} c_{11} + x(k, b, z) \mathbf{1}_{\{\Delta=1\}} \min\{c_7, s_7 k\} \right\} \Gamma(dk, db, z), \quad (35)$$

and entrants' investment and costs are

$$E = M \{ k'_E + \kappa + \lambda_E (-k'_E + q(k'_E, b'_E) b'_E - \kappa) \}. \quad (36)$$

5 Calibration

In this section we present the calibration of the model. A summary of the model implied definitions for key variables we observe in the data is given in Table 2.

Table 2: Model Definitions

Variable	Model Expression
Book Value Assets	$k + I_{\{b < 0\}}(-b)$
Capital	k
Net Debt	b
Total Debt	$I_{\{b \geq 0\}}b$
Operating Income	$\pi = zk^\alpha n^{1-\alpha} - wn - c_f$
Taxable Income	$\pi - \delta k - \left(\frac{1}{q} - 1\right) \frac{b'}{(1+r)}$
Cash Flow	$\pi - \left(\frac{1}{q} - 1\right) b' - T$
Equity Issuance	$I_{\{e < 0\}}e$
Dividends	$I_{\{d \geq 0\}}(1 - \tau^d)d$
Gross Investment	$i^g = k' - (1 - \delta)k$
Net Investment	$i^n = i^g - \delta k$
Secured Debt	$I_{\{b \geq 0\}} \min\{sk, b\}$
Unsecured Debt	$I_{\{b \geq 0\}} \max\{0, b - sk\}$
Market Value Assets	$V(k, b, z) + qb'$

A period is set to be one year. Conditional on issuing a positive amount of equity, we parameterize equity issuance costs for non-bankrupt firms as a quadratic function of the following form:

$$\lambda(x) = \lambda_0 + \lambda_1 |x| + \lambda_2 x^2 \quad (37)$$

Thus, when $\lambda_0 > 0$ there include fixed costs to issuing equity. The same functional form is used for firms going into chapter 11 and entrants. To reduce the number of parameters to be estimated we set $\lambda_0^{11} = \lambda_0$, $\lambda_2^{11} = \lambda_2$, $\lambda_1^E = \lambda_1$ and $\lambda_0^E = \lambda_2^E = 0$.¹⁴

We assume that firm productivity follows an AR(1) process

$$\log(z_t) = \rho_z \log(z_{t-1}) + \epsilon,$$

with $|\rho_z| < 1$ and $\epsilon \sim N(0, \sigma_\epsilon)$. We use Tauchen's procedure to discretize this process into a 11 state Markov process $\{z_1, \dots, z_{11}\}$.

In summary, our model has 22 parameters to be calibrated which appear in Table 3. We divide them in two sets. The first set (those above the line in Table 3) are calibrated outside the model using standard values in the literature or independent targets. Note that once we set the pre-tax risk-free rate, together with income tax τ_i , equilibrium conditions determine β , r and q^B . More specifically, $\beta = q^B = (1 + r)^{-1} = \tilde{q}^B * (1.0 + \tau_i) - \tau_i$. To estimate the parameters of the z process we follow Cooper and Haltiwanger [13]. In particular, taking logs

¹⁴The equity issuance cost function for entrants boils down to a linear function of x . This greatly simplifies the computational algorithm without affecting the quantitative results of the model.

of operating income (evaluated at optimal labor) and quasi-differencing yields

$$\pi_{it} = \rho_z \pi_{it-1} + \theta k_{it} - \rho_z \theta k_{it-1} + \epsilon_{it}, \quad (38)$$

where $\theta = \frac{\alpha\nu}{1-(1-\alpha)\nu}$. We estimate this equation for firms outside bankruptcy using a panel fixed effect estimator with a complete set of dummies to capture year fixed effects. The results provide us with an estimate of ρ_z and σ_ϵ .

The parameters below the line in Table 3 are set by minimizing the distance between model moments and data moments selected to provide identification of the model parameters. Specifically, the parameters are chosen to minimize

$$J(\Theta) = [\mu^d - \mu^s(\Theta)]' W^* [\mu^d - \mu^s(\Theta)] \quad (39)$$

with respect to parameters Θ , where μ^d are the moments from the data, $\mu^s(\Theta)$ are the moments from the simulated model at parameters Θ and W is a positive definite weighting matrix.¹⁵

Table 3 presents the parameter values.

¹⁵In a first pass, we set W to the identity matrix. We then estimate the optimal weighting matrix from the inverse of the variance-covariance matrix of the simulated moments.

Table 3: Parameter Values

Parameter		Value	Targets
Discount Rate	r	0.020	T-Bill Rate
depreciation rate	δ	0.150	Capital Dep. Rate Compustat
capital share	α	0.330	standard parameter
returns to scale	ν	0.850	standard parameter
Corporate Tax Rate	τ_c	0.30	Hennessy - Whited [18]
Dividend Tax Rate	τ_d	0.12	Hennessy - Whited [18]
Income Tax Rate	τ_i	0.25	Hennessy - Whited [18]
Debt Cost in Chapter 11	λ_b^{11}	1.000	
autocorrelation z	ρ_z	0.649	Autocorrel Op. Inc. (eq. (38))
Std. Dev. Shock	σ_ϵ	0.199	Std. Dev. Op. Inc. (eq. (38))
Fixed Cost Production	c_f	0.590	bankruptcy rate
Chapter 7 cost	c_7	0.001	fraction of exit by Ch 7
Price capital after liquidation	s_7	0.299	recovery rate Ch 7
Chapter 11 cost	c_{11}	0.200	fraction of bankruptcy Ch 11
Firm's Bargaining Power	θ	0.353	recovery rate CH 11
Equity Issuance Cost	λ_0	0.020	Frac. Firms issuing equity
Equity Issuance Cost	λ_1	0.201	Avg. equity issuance Non-Bankrupt
Equity Issuance Cost	λ_2	0.025	Variance equity issuance
Equity Issuance Cost Ch 11	λ_1^{11}	0.287	Debt to Assets Ch 11
Adjustment Cost	ψ	0.251	Net Investment Non-Bankrupt
Entry cost	κ	10.688	entry / exit rate
			Debt to Assets Non-Bankrupt

Note: Parameters below the line are set by minimizing the distance between model moments and target moments. We set $\lambda_0^{11} = \lambda_0$, $\lambda_2^{11} = \lambda_2$, $\lambda_1^E = \lambda_1$ and $\lambda_0^E = \lambda_2^E = 0$.

Data comes from Compustat and was presented in Section 2. Before presenting the estimation outcome, we discuss the selection of these moments. Since every moment that results from the model is a function of all parameters, there is no one-to-one link between parameters and moments. However, we can point to moments that are more informative to pin down a given parameter or set of parameters than others. The value of the fixed operating cost c_f is important for matching the bankruptcy rate. The cost of filing for chapter 7 bankruptcy is important for matching the fraction of exit by Chapter 7 bankruptcy. The observed recovery rate for firms that go into Chapter 7 contains information helpful in choosing the value of capital after liquidation s_7 . The cost of filing for Chapter 11 bankruptcy is important for matching the fraction of Chapter 11 bankruptcy. The bargaining power of the firm once in reorganization is important for matching the observed recovery rate in Chapter 11. The equity issuance cost parameters affect the borrowing decisions of the firm directly so the fixed cost λ_0

is selected to help match the fraction of firms in any given year issuing equity, the linear cost λ_1 to match the average equity issuance by non-bankrupt firms and the quadratic parameters λ_2 to match the variance of equity issuance for non-bankrupt firms. The debt-to-asset ratio for non-bankrupt firms is also informative for these parameters. The differential cost of equity issuance for firms going into Chapter 11 λ_1^{11} is selected to match the debt-to-asset ratio of firms in Chapter 11. The average of the productivity process helps match firms' size (number of workers) in the model to that in the data. The net investment rate provides information on the adjustment cost parameter ψ . Finally, the entry cost is selected to match the entry rate (that in steady state is equal to the exit rate).

Table 4: Comparison of Data and Model Moments

	Benchmark	
	Data	Model
Frequency of all bankruptcy %	0.87	0.77
Fraction of Exit by Chapter 7 %	59.23	52.63
Recovery Rate in Chapter 7 %	27.4	28.5
Fraction of Bankruptcy by Chapter 11 %	80.82	63.43
Recovery Rate in Chapter 11 %	69.40	78.38
Frac. Firms issuing equity	27.63	55.77
Equity Issuance / Assets non-Bankrupt %	8.1	14.19
Variance equity issuance %	7.44	9.7
Debt / Assets Chapter 11 %	79.51	107.71
Net Investment / Assets non-bankrupt %	1.43	3.45
Entry/Exit Rate %	0.71	0.54
Debt / Assets non-Bankrupt %	57.38	46.46
Net Investment / Assets Ch 11 %	-4.37	-60.47
Equity Issuance / Assets Chapter 11 %	4.83	56.16
Net Debt / Assets non-Bankrupt %	26.08	35.38
Net Debt / Assets Chapter 11 %	58.05	107.71
Dividend/Assets non-Bankrupt %	5.92	2.04
Dividend / Assets Chapter 11 %	3.35	0.00
Interest Rate Non-Bankrupt %		2.79
Interest Rate Chapter 11 %		12.12
Avg Size (k) / Prod. z Non-Bankrupt		9.57 / 1.44
Avg Size (k) / Prod. z Ch 11		4.40 / 0.91
Avg Size (k) / Prod. z Ch 7		0.54 / 0.69

Given these parameter values, the moments we find in the model are given in Table 4. In general, the model does a good job approximating the targeted moments but overestimates the fraction of firms issuing equity, the equity issuance over assets of non-bankrupt firms as

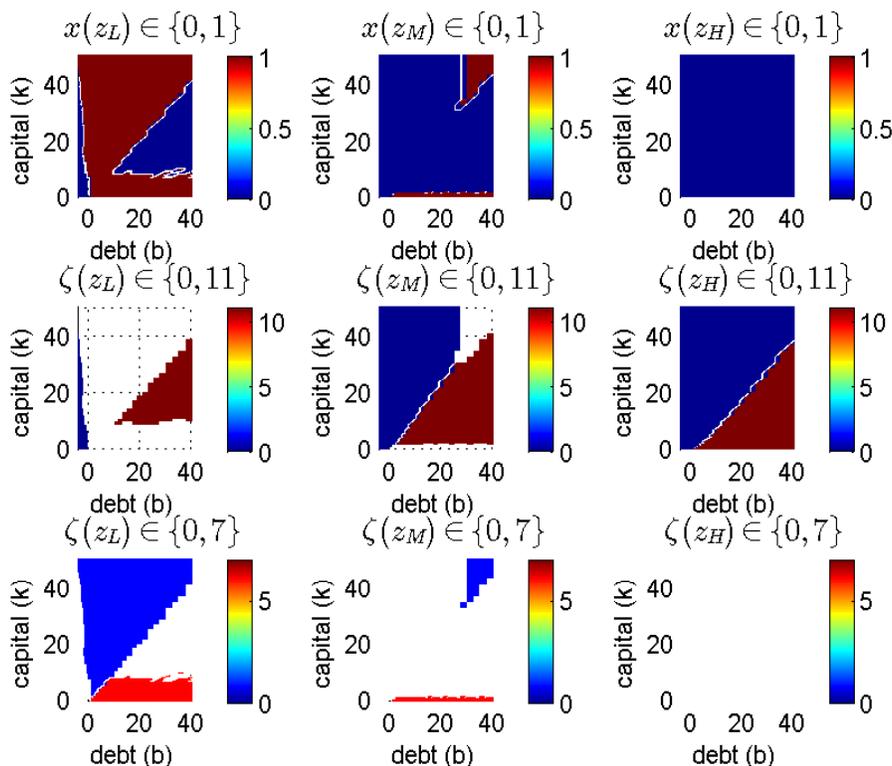
well as the debt to asset ratio of Chapter 11 firms. Among those moments that were not targeted the model generates a much larger (in absolute terms) net investment rate and equity issuance over assets for firms in Chapter 11 and underestimates the dividend ratio for both non-bankrupt and Chapter 11 firms.

6 Results

6.1 Positive Analysis

We begin by describing decision rules concerning exit and bankruptcy choice. Figure 3 presents the exit and default decision rules across capital, debt and productivity. The left panels present exit (top), reorganization (i.e. whether to repay in full or go to Chapter 11, middle), and liquidation (i.e. whether to exit by full repayment or go into Chapter 7 bankruptcy, bottom) decision rules respectively for a firm with low productivity ($z = z_L$). The next two columns present the same decision rules for firms with median and high productivity ($z = z_M$ and $z = z_H$). The blank regions in the middle and bottom panels occur when the decision rule is not relevant. More specifically, in states where a firm chooses to exit $x(k, b, z) = 1$ the decision rule for reorganization is not shown. Similarly, in states where a firm chooses to stay $x(k, b, z) = 0$ the decision rule for liquidation is not shown.

Figure 3: Exit and Default Decision Rules



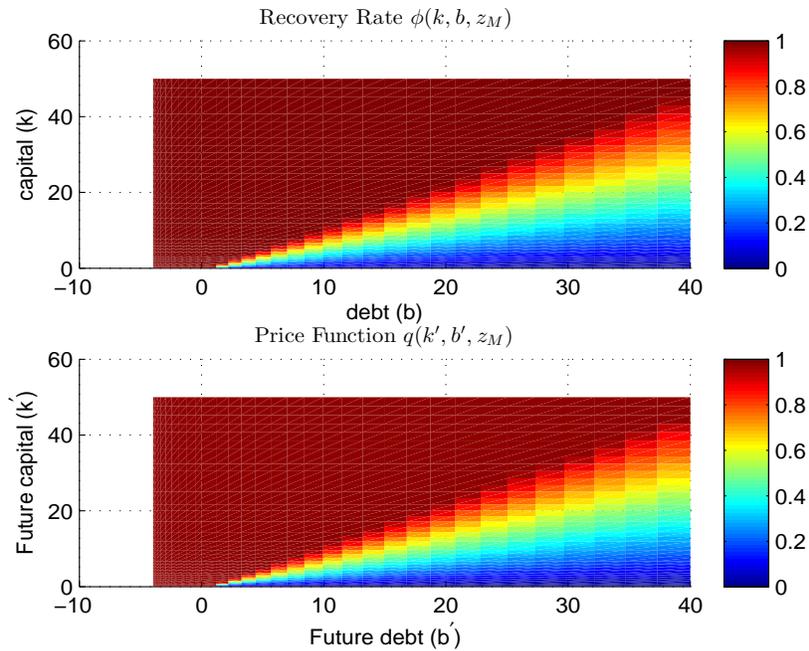
As evident in Figure 3 firms with high productivity (the rightmost column) do not exit no matter what their mix of capital and debt. Some firms with high productivity and high debt do however choose Chapter 11. At the other end of the spectrum (the leftmost column), firms with low productivity: (i) negative net debt (i.e. cash) and low capital, choose to continue operating, (ii) low debt and high capital choose to exit, (iii) high debt and low capital choose Chapter 7; and (iv) high debt and medium amounts of capital choose Chapter 11. Qualitatively, median productivity firms behave more like low productivity firms (since all options are chosen) than high productivity firms.

We next describe borrowing rates offered to firms conditional on how much they borrow (b'), what collateral they will have next period when they have to repay (k'), and their current productivity (z) proxying for their productivity. Figure 4 graphs equilibrium price menus offered to firms with median productivity (top panel) and low productivity (bottom panel). As evident in Figure 4, for a given level of capital (which serves as collateral), the higher a firm's debt the less lenders recover and for a given level of debt, the higher a firm's capital the more lenders recover.¹⁶ Thus, firms with high debt to assets face higher interest rates. Further

¹⁶We note that at high debt and capital levels, the recovery rate is 100% since at those levels the participation

note that for a given level of borrowing and collateral (b', k') , prices for firms with median productivity (top panel) are uniformly higher than those offered to low productivity firms (bottom panel). While not pictured, highest productivity firms actually borrow at the risk free rate for almost all levels of (b', k') . Thus, more productive firms are offered lower interest rates than less productive firms. Finally, note that equilibrium interest rates observed in the economy depend not only on these menus but also the equilibrium cross-sectional distribution of firms. Table 4 makes clear, for instance, that the equilibrium average interest rate which non-bankrupt, non-exiting firms face is lower (2.79%) than those faced by firms which are reorganizing (12.12%).

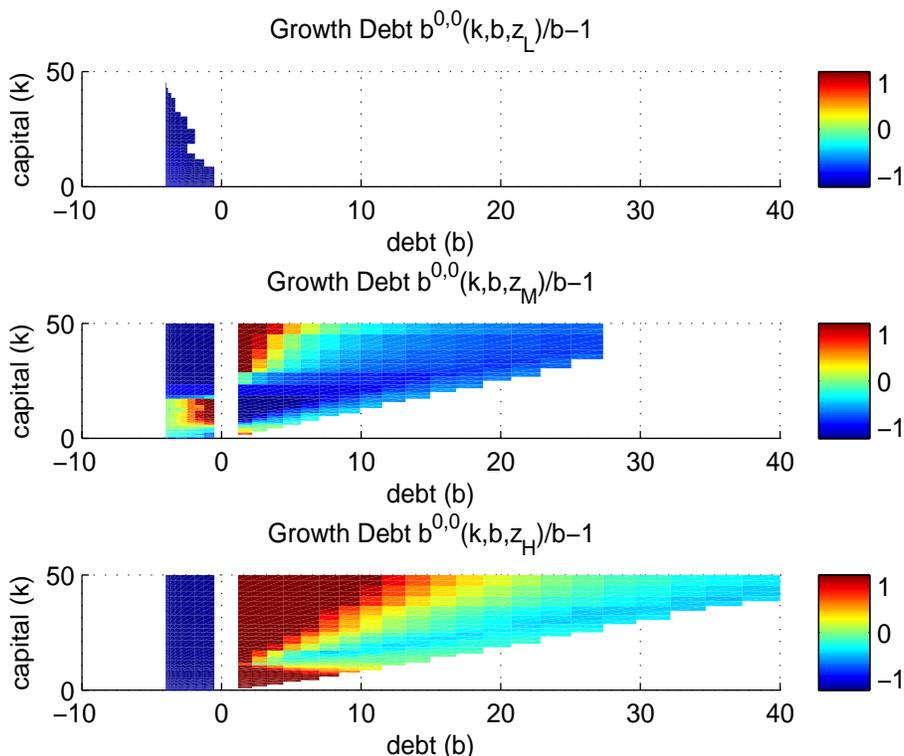
Figure 4: Debt Price Schedules



Debt dynamics for Nonbankrupt/Non-exiting ($\Delta = 0, x = 0$) firms are illustrated in Figure 5. Again, it should be noted that white sections across the state space imply that at that state, firms choose something other than $(\Delta = 0, x = 0)$. Low productivity firms which are nonbankrupt/non-exiting choose to accumulate debt (since by Figure 3 we know only firms with negative net debt choose $(\Delta = 0, x = 0)$ and since they start with negative debt, the change is negative due to the denominator). For median and high productivity firms we find they choose (i) to accumulate debt to finance positive net investment if they start with low debt and high capital (since the high capital serves as collateral and so they face low interest rates) and (ii) lower their debt holdings at high debt to capital ratios.

constraint of the firm (20) is violated.

Figure 5: Change in Debt Non-Bankrupt



Dividend distribution ($e > 0$) and equity issuance ($e < 0$) is illustrated in Figure 6. Firms with low productivity who do not exit and are liquid (i.e. have net cash holdings $b < 0$) choose to pay dividends. Firms with median and high productivity, choose to pay dividends if they have low debt and high capital and issue equity if they have high debt and low capital (equity issuance is primarily done by the median type since they face high interest rates at high debt/capital).

The equilibrium conditional distributions of firms are illustrated in Figure 7. It is evident that firms with low productivity are amassed on lower capital and debt levels while those with high productivity are amassed on higher capital and debt levels.

6.2 Normative Analysis

In our counterfactual experiment, we analyze a variant of the bankruptcy procedure proposed by Aghion, Hart and Moore [4], which itself is related to Bebchek [6]. In particular, their proposal consists of three simple steps: (i) When a firm goes bankrupt, a fraction $(1 - \varphi)$ of the firm's existing debts are canceled (Aghion, et. al. take $\varphi = 0$); (ii) bids are solicited for the "new", all equity firm and rights to the equity in this new firm are allocated among

Figure 6: Net cash flow over Assets Non-Bankrupt

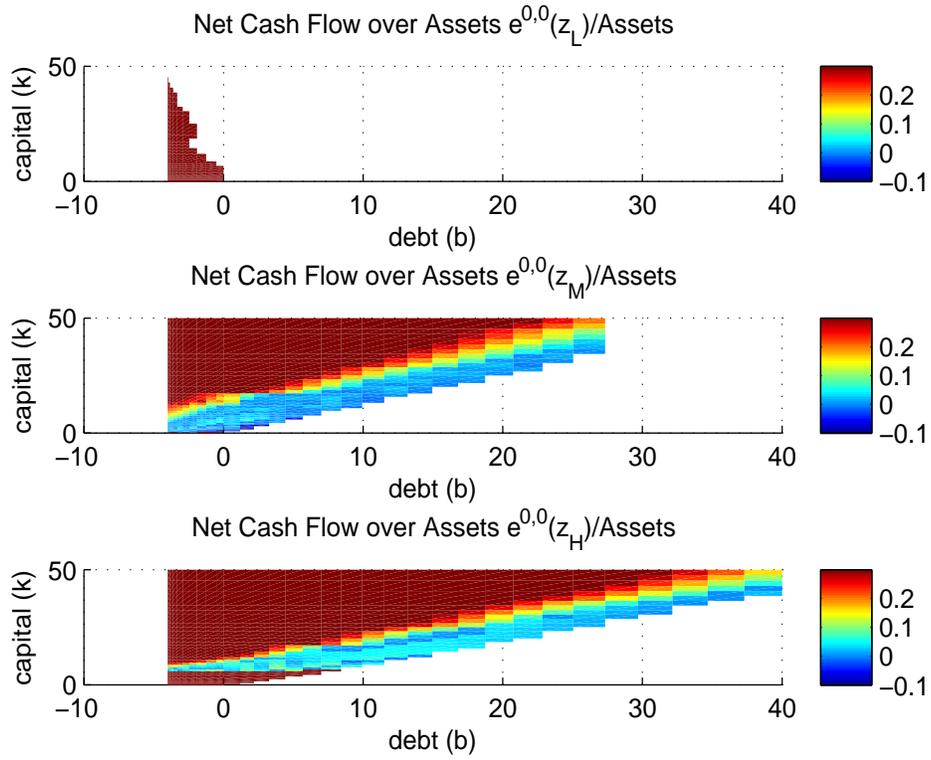
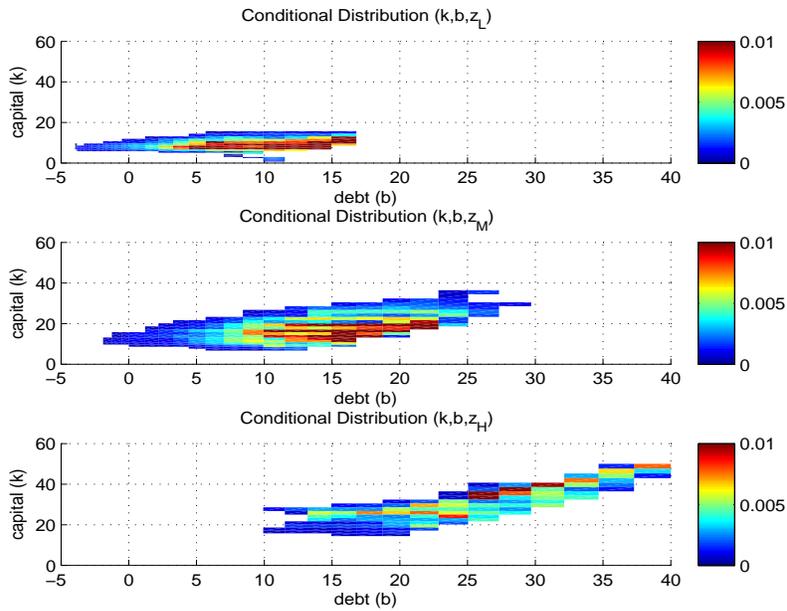


Figure 7: Distribution of Firms (conditional on z)



the former claim holders (applying absolute priority rule, first to bond holders, then to former equity holders);¹⁷ (iii) the new shareholders –that is, the former claim holders– decide whether to continue the all equity firm or exit. After these steps the firm exits from bankruptcy. A similar proposal has recently been suggested by the American Bankruptcy Institute.¹⁸

At the beginning of the period, the firm decides whether to declare bankruptcy ($\Delta = 1$) or not ($\Delta = 0$). If it decides not to default, the firm repays its debt and decides whether to continue ($x = 0$) or not ($x = 1$). If it decides to default, the procedure described above is triggered.

As before, the value of the firm is given by

$$V(z, k, b) = \max_{\Delta \in \{0,1\}} V_{\Delta}(z, k, b) \quad (40)$$

where as before

$$V_{\Delta=0}(z, k, b) = \max_{x \in \{0,1\}} V_0^x(z, k, b) \quad (41)$$

but now $V_{\Delta=1}(z, k, b)$ must be consistent with the process outlined in the new bankruptcy procedure.

Specifically, a firm which does not declare bankruptcy and chooses not to exit will decide its optimal level of investment and borrowing as before:

$$V_0^0(z, k, b) = \max_{n \geq 0, k' \geq 0, b'} \left\{ d + (1+r)^{-1} E_{z'|z} [V(z', k', b')] \right\} \quad (42)$$

s.t.

$$e = \pi - T^c(k, z, k', b') - (k' - (1 - \delta)k) - b + q(b', k', z)b' - \Psi(k', k)$$

$$d = \begin{cases} (1 - \tau_d)e & \text{if } e \geq 0 \\ e - \lambda(e) & \text{if } e < 0. \end{cases}$$

Further, in the event of exiting without default, the dividend policy implies as before

$$V_0^1(z, k, b) = \begin{cases} (1 - \tau_i)(k - b) & \text{if } k \geq b \\ k - b - \lambda(k - b) & \text{if } k < b \end{cases} . \quad (43)$$

Unlike before, in the event the firm declares bankruptcy, given limited liability its value is given by:

$$V_1(z, k, b) = \max \{W(z, k, \varphi b) - b - c_B, 0\}, \quad (44)$$

¹⁷The solicitation of bids means the firm is competitively priced in our framework. More generally, multiple bids can help in environments with private information.

¹⁸Consistent with the policy recommendation in Aghion, et. al. [4], Stech [25] writes about the ABI proposal “Under the proposed changes, a bankruptcy judge would estimate the reorganization value of a company once it files for Chapter 11 protection. The bankrupt company could reorganize and leave Chapter 11 by paying that amount to a senior lender, even if that lender’s debt has a larger face value.” For the full report see ABI [2].

where c_B is the bankruptcy cost and $W(z, k, \varphi b)$ is the value of the “new” firm after a fraction $1 - \varphi$ of its original debts have been canceled. This value captures the fact that the new owners of the firm have the option to either continue operating the firm or liquidate it and is given by

$$W(z, k, \varphi b) = \max\{V(z, k, \varphi b), sk\}. \quad (45)$$

If $\varphi = 0$ as in Aghion, et. al. then we can replace $V(z, k, 0)$ in (45) with $V_0(z, k, 0)$ since a firm without debt cannot declare bankruptcy because there is nothing to default on.

One important aspect of this counterfactual is the pricing of debt. The bankruptcy set is given by

$$D_1(k, b) = \{z \in Z : \Delta(z, k, b) = 1\}.$$

In this case, we can write the lender’s profit function as follows:

$$\begin{aligned} \Omega(b', k', z) &= -q(b', k', z)b' + q^B[1 - \Lambda(b', k', z)]b' \\ &+ q^B \sum_{z' \in D_1(k', b')} \min\{b', \max\{W(z', k', \varphi b') - c_B, 0\}\}G(z'|z). \end{aligned} \quad (46)$$

In equilibrium, lender’s profits must be zero.

Aggregate consumption in this economy is again given by the resource constraint (29) where Y , CF , I and E are defined as before in (30), (31), (32), (36) while aggregate equity issuance costs are¹⁹

$$\Lambda = \int_{K \times B} \sum_z [(1 - \Delta(k, b, z)) (\mathbf{1}_{\{e < 0\}} \lambda(e)) + \Delta(k, b, z) (\mathbf{1}_{\{V(z, k, \varphi b) > sk, e < 0\}} \lambda(e))] \Gamma(dk, db, z),$$

final distributions from exiting firms are

$$X = \int_{K \times B} \sum_z x(k, b, z) \{ \mathbf{1}_{\{\Delta=0\}}(k - b) + \mathbf{1}_{\{\Delta=1\}} \max\{W(z, k, \varphi b) - b - c_B, 0\} \} \Gamma(dk, db, z),$$

and aggregate bankruptcy costs are

$$BC = \int_{K \times B} \sum_z x(k, b, z) \mathbf{1}_{\{\Delta=1\}} \min\{c_B, sk\} \Gamma(dk, db, z).$$

Notice that the key essential difference between the proposed bankruptcy reform and that of the current law is that absolute priority rule is applied in all cases in the proposal while it

¹⁹To save on notation, the first expression uses the fact that a firm will never pay back debt and issue equity when it exits.

is only applied in Chapter 7 currently. This can have a big impact on recovery rates and the pricing of debt. In particular, in return for debt forgiveness the creditors receive an all-equity firm without having to go through a bargaining process.²⁰ This alters recovery rates in (46) and hence interest rate menus that firms face. This will have important implications for firm dynamics.

In our main experiment, we set $\varphi = 0$ as proposed by Aghion, Hart and Moore [4] and the bankruptcy costs equal to those under Chapter 11 bankruptcy in our benchmark economy (i.e. $c_B = c_{11}$). Table 5 presents a comparison across the two steady states.

²⁰For all the benchmark parameters in Table 3, we have done a comparative static exercise of changing bargaining weight θ . In our benchmark, $\theta = 0.35$ yields recovery $\phi = 0.78$. As the firm gains bargaining power $\theta = 0.99$ recovery falls to $\phi = 0.5$ and as the lender gains power $\theta = 0.01$ recovery rises to $\phi = 0.88$ but not to total forgiveness in Chapter 11.

Table 5: Counterfactual New Bankruptcy Procedure

	Data	Benchmark Model	Bankruptcy Reform
Frequency of all bankruptcy %	0.87	0.77	4.74
Fraction of Exit by Liquidation %	59.23	52.63	22.82
Recovery Rate in Liquidation %	27.4	28.5	0.08
Fraction of Bankruptcy Reorganization %	80.82	63.43	98.48
Recovery Rate in Reorganization %	69.4	78.38	69.93
Frac. Firms issuing equity	27.63	55.77	12.26
Equity Issuance / Assets non-Bankrupt %	8.1	14.19	0.59
Variance equity issuance %	7.44	9.7	0.09
Debt / Assets Reorganization %	79.51	107.71	165.17
Avg. Number workers (000s)	1.85	3.82	3.72
Net Investment / Assets non-bankrupt %	1.43	3.45	4.45
Entry/Exit Rate %	0.71	0.54	0.31
Debt / Assets non-Bankrupt %	57.38	46.46	79.96
Net Investment / Assets Reorganization %	-4.37	-60.47	-14.28
Equity Issuance / Assets Reorganization %	4.83	56.16	3.71
Net Debt / Assets non-Bankrupt %	26.08	35.38	69.79
Net Debt / Assets Reorganization %	58.05	107.71	165.17
Dividend/Assets non-Bankrupt %	5.92	2.04	4.60
Dividend / Assets Reorganization %	3.35	0.00	3.11
Interest Rate Non-Bankrupt %		2.79	3.77
Interest Rate Chapter Reorganization %		12.12	4.26
Avg Size (k) / Prod. z Non-Bankrupt		9.57 / 1.44	9.39 / 1.44
Avg Size (k) / Prod. z reorganize		4.40 / 0.91	2.17 / 0.97
Avg Size (k) / Prod. z Liquidation		0.54 / 0.69	0.55 / 0.59
Avg Size (k) entrant		8.10	11.54

Notes: In the benchmark, liquidation (Chapter 7) refers to $(\Delta = 1, x = 1)$ while reorganization (Chapter 11) refers to $(\Delta = 1, x = 0)$. In the reform, liquidation refers to $\Delta = 1$ and $V(z, k, 0) < sk$ while reorganization refers to $\Delta = 1$ and $V(z, k, 0) \geq sk$.

To understand the differences between the benchmark and reform economies, we begin by comparing the interest rate menus in both cases. As we discussed before, the reform allows firms to access better credit terms and reduces the need for the firm to hold as much capital (collateral against loans). Figure 8 presents the price schedule for both cases (Panel (i) our benchmark and Panel (ii) the counterfactual economy).

Figure 8: Comparison Equilibrium Price Function

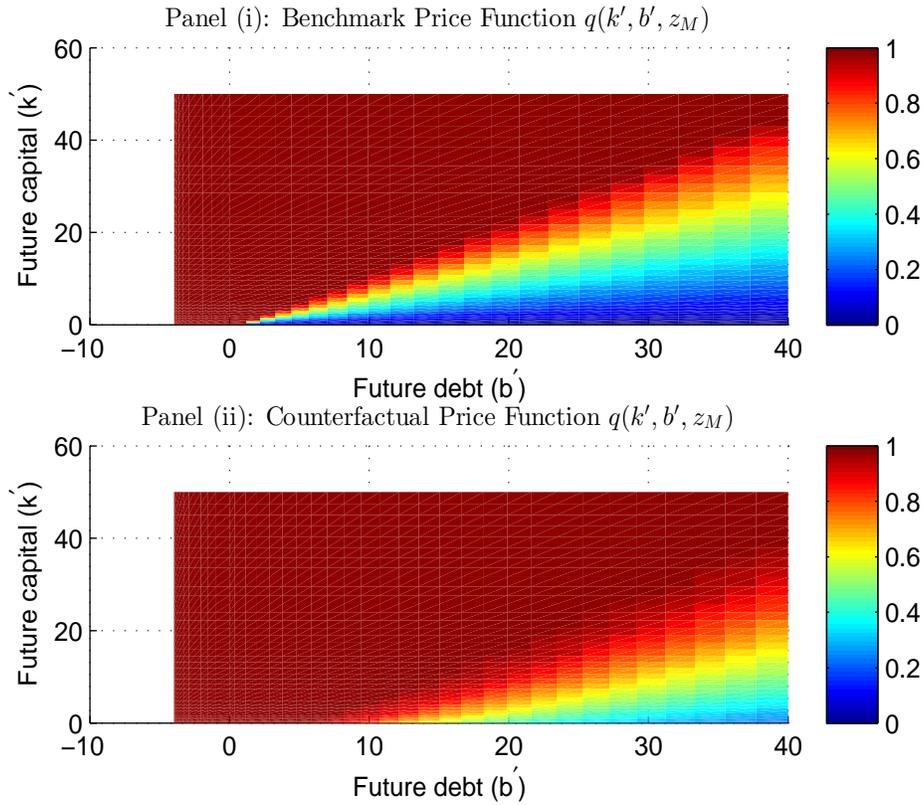


Figure 8 makes clear that at any given level of (b', k') , prices are weakly higher in the reform economy than in our benchmark. This derives from the fact, with a more “lender” friendly economy, at a given $\frac{b'}{k'}$, the expected recovery rate goes up (and in fact is increasing in $\frac{b'}{k'}$), allowing the firm to borrow at lower interest rates ceteris paribus.

The resultant shift in interest rate menu has two effects: (i) firms are willing to borrow more in order to invest (potentially getting close to where marginal product of capital equals the risk free rate); and (ii) they need to hold less collateral in order to sustain a given level of investment. These two effects are easily observed in Figures 9 and 10 which graph the distributions of debt to assets and the distribution of capital to assets respectively.

Figure 9: Distribution of Debt to Assets

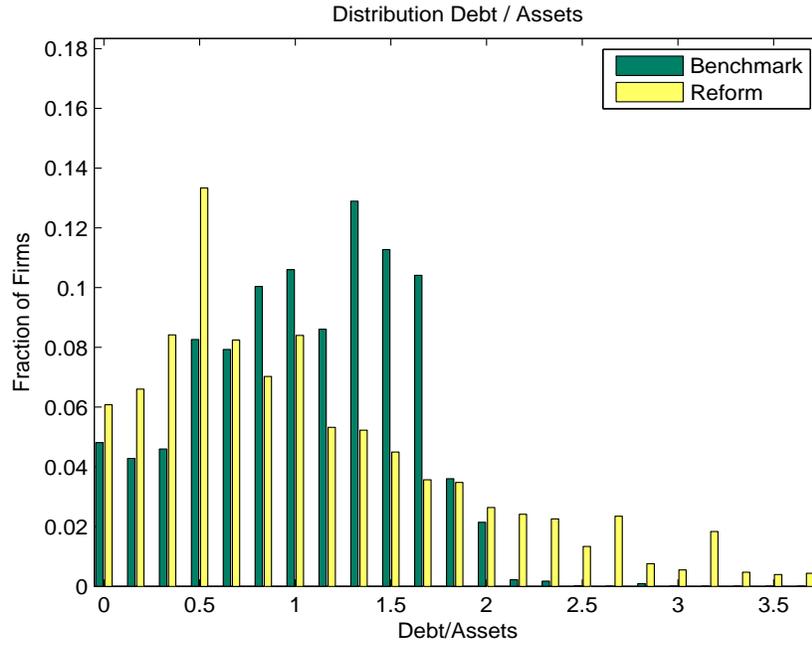
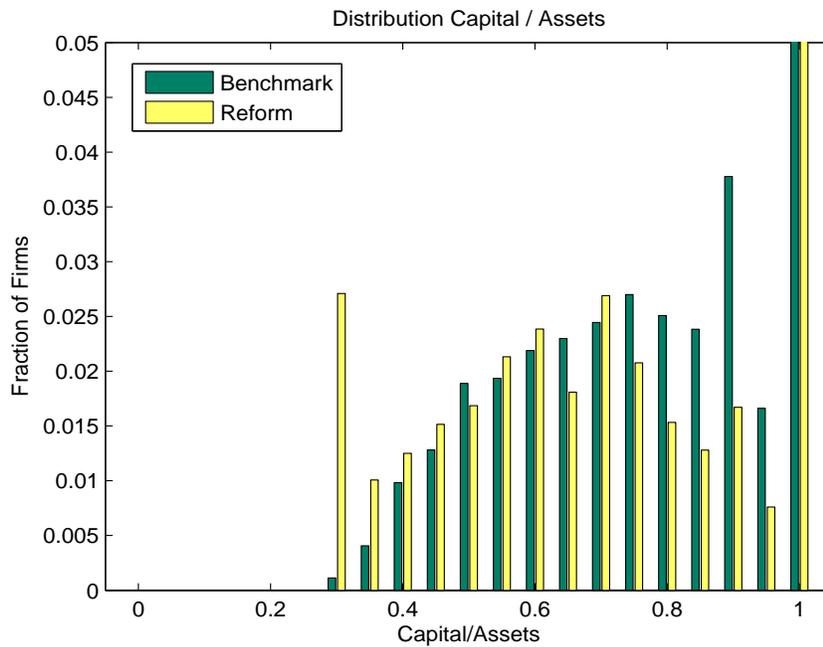


Figure 10: Distribution of Capital to Assets



Note: in order to focus on positive cash holdings, we truncated the frequency of the value of capital to assets at 1, which corresponds to zero cash holdings. The mass of firms with capital to asset equal to 1 in the figure is 73.43% and 75.50% in the benchmark and the reform economy respectively.

Consistent with the preceding two figures, Table 5 documents a 72% increase in borrowing (relative to assets) and sharp reduction in equity issue (relative to assets decreases almost 100%) by non-bankrupt firms. This increase in borrowing results in a higher realized average interest rate for non-bankrupt firms (+35%) from from 2.79% to 3.77%. Bankrupt firms also borrow more (debt to assets increases 56%) and reduce equity issuance dramatically (a 93% reduction) in the reform economy. The reform drastically lowers the realized average interest rate of bankrupt firms by 65% from 12.12% to 4.26%. These capital structure decisions support higher net investment to assets; on average, a 29% increase for non-bankrupt firms and 76% decrease in negative net investment by bankrupt firms.

Table 5 also documents that while the fraction of firms declaring bankruptcy rises (by more than 5 times), the fraction of firms exiting and those that exit by liquidation following bankruptcy falls sharply (-43% and -57% respectively). Firm reorganization after bankruptcy (via ownership change and debt forgiveness) rises considerably (55%). The total mass of firms depends directly on the fraction of entrants that survive over time. While the reform economy exhibits a lower mass of entrants M (29 percent smaller), the reduction in exit rates results in a higher total mass of firms after the reform than in the benchmark (an increase of 8 percent). This increase in the total mass of firms leads to an increase of aggregate capital by 2.88% in the reform economy despite the leftward shift in the distribution of capital to assets that we observed in 10.

The effects of the reform on the life-cycle of firms can be computed for a given cohort of firms.²¹ Figure 11 shows the survival, exit and bankruptcy probabilities for a given cohort of firms under the benchmark and after the bankruptcy reform. This figure makes evident that a firm is less (more) likely to survive or declare bankruptcy early (late) in its lifecycle. Since firms enter bigger in the reform economy, if they experience a low productivity draw they are more likely to exit by selling off their capital. The increase in the bankruptcy rate after initial entry is the result of higher borrowing. Obviously, the long run behavior of firms is determining the averages given in Table 5 (i.e. lower exit and higher bankruptcy long run averages). This implies that in a steady state average age is higher in the reform economy than in the benchmark.

²¹To track the evolution of a given cohort of firms, we start with a mass of firms M and follow them over time (age) using equation (27) with no entry after the initial period. Statistics presented are conditional on firm age.

Figure 11: Survival, Exit and Bankruptcy by Firm Age

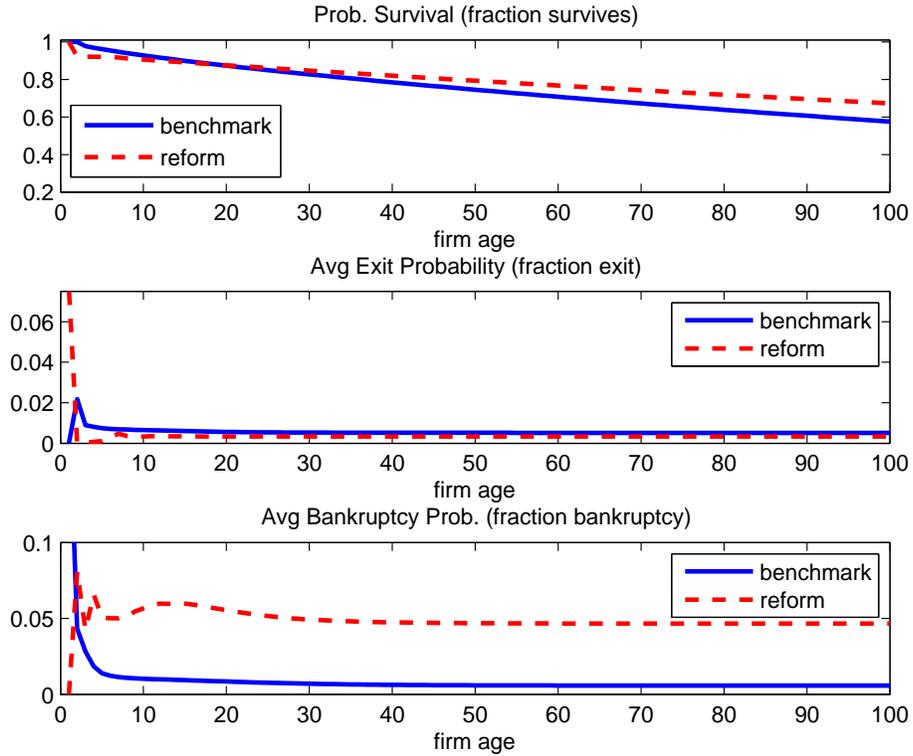
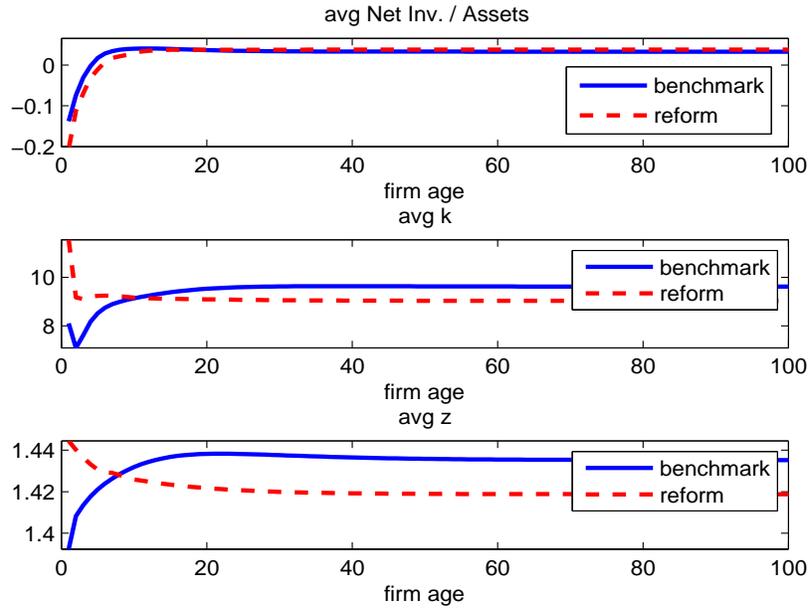


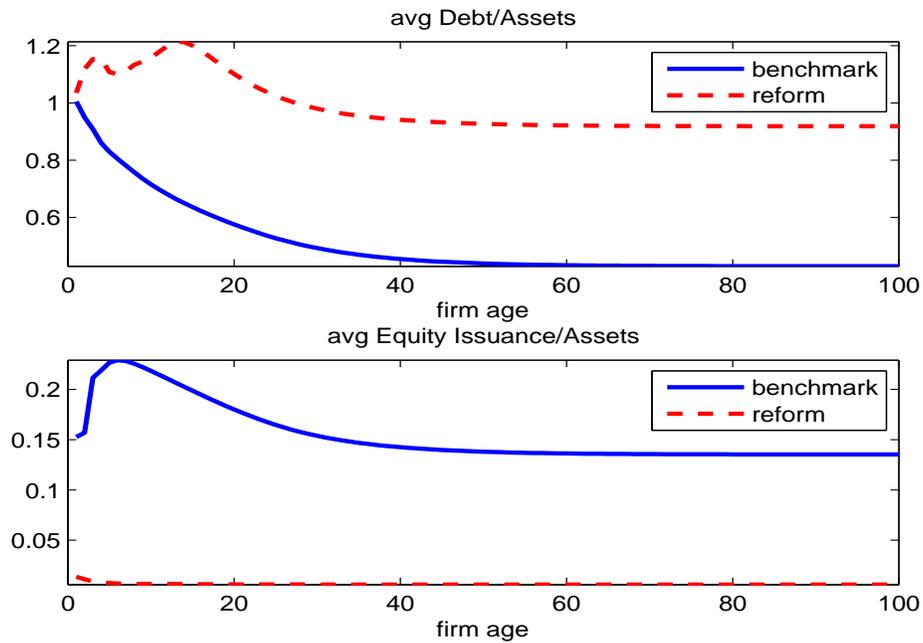
Figure 12 shows the evolution of net investment relative to assets, firm size (capital) and productivity over the firm's life cycle. Entrants' size rises by 42%, as well as the size of young incumbent firms ($\text{age} \leq 7$ years) and their productivity. While the stochastic process for technology shocks does not change across experiments, Figure 12 shows that after entry, it is more likely for young, less productive firms to survive under the benchmark. Incentives to exit or go bankrupt are stronger for firms with low productivity and a high leverage ratio (b/k). Since firms are offered better credit terms after the reform, entrants are able to borrow more and as we showed in figure 12 leverage ratios are higher after the reform, selection effects result in higher average productivity for young firms in the economy with the new bankruptcy regime. These effects are reversed when old (since firms previous to the reform slowly grow and accumulate capital).

Figure 12: Evolution of Net Investment, Firm Size and Productivity



The capital structure decisions which support the above investment choices are given in Figure 13. The figure shows that mature firms sustain a higher debt to asset ratio and lower equity issuance to asset ratio along their life cycle.

Figure 13: Debt to Assets and Equity Issuance to Assets over age



The positive effects in Table 5 have implications for household welfare (aggregate consumption), which is presented in Table 6.²² The table shows that the increase in investment (+3.05%), fixed costs (+8.44%), adjustment costs (+4.38%), and bankruptcy costs together with the reduction in exit value (-50.85%) after the reform are more than compensated by the increase in output (+1.99%), the reduction in equity issuance costs (-71.28%) and entry costs (-14.09%) that result in an increase in aggregate consumption (2.32%). The increase in output and investment are a result of the increase in the level of productivity due to selection effects (as we discussed above we observe higher exit and bankruptcy rates) as well as the reduction in cost of borrowing. Better credit terms are also the driver of lower equity issuance (reducing the need to issue equity in order to finance investment) plus, since after the reform firms that reorganize change owners, they do not need to either issue debt or equity in order to pay for bankruptcy costs (paid for by the previous owners). The final important component in explaining the welfare result is the change in entry costs. Their reduction is mostly due to the reduction in the entry rate (since it is equal to the exit rate) pushing the aggregate entry cost lower even though the size of the entrant is higher (since borrowing cost for entrants are lower under the reform)

²²Because we consider steady state equilibria and we have a representative household, there are no transition welfare effects.

Table 6: Bankruptcy Reform: Welfare and Aggregates

	Benchmark Model	Bankruptcy Reform	Change (%)
Aggregate Consumption C	1.156	1.183	2.32
Aggregate Output Y	1.745	1.779	1.99
Fixed Cost CF	0.153	0.166	8.44
Investment I	0.3613	0.3723	3.05
Equity Issuance Λ	0.023	0.007	-71.28
Exit Value X	0.001	0.000	-50.85
Bankruptcy Costs BC	0.0003	0.0026	892.31
Entry Costs E	0.023	0.020	-14.09
Adjustment Costs Ψ	0.028	0.029	4.38
Equilibrium wage	1.00	1.013	1.34
Capital to output ratio K/Y	1.42	1.43	0.87
Measured TFP ($= Y/K^{1/3}$)	1.29	1.30	1.03
Avg. Productivity \bar{z}	1.96	1.97	0.41
Avg. (output weighted) Prod. \hat{z}	1.43	1.43	-0.16
$Cov(z, \omega)$	0.53	0.54	1.95

Note: \bar{z} is average firm productivity, \hat{z} is the (output weighted) average firm level productivity and ω is the output share of each firm.

We believe an important aspect of the bankruptcy reform is that it improves the allocation of resources in the economy. Table 6 shows that measured aggregate total factor productivity (TFP) rises by more than 1%. In order to explain this change and to provide a measure that captures how efficiently resources are allocated in the economy we use a decomposition of weighted average firm level productivity proposed originally by Olley and Pakes [24]:

$$\hat{z} \equiv \int_{K \times B} \sum_z z_j \omega_j dj = [\bar{z} + cov(z, \omega)]$$

where \hat{z} is the average of firm level productivity weighted by output share, ω_j is the output share of each firm j , and \bar{z} is the unweighted mean productivity (i.e., $\int_{K \times B} \sum_z z \Gamma(dk, db, z)$). That is, output weighted productivity can be decomposed into two terms: the unweighted average of firm-level productivity plus a covariance term between output shares and productivity. A larger value for the covariance term captures an improvement in allocative efficiency. Table 6 shows the values for this decomposition. We observe that while average (un-weighted) productivity decreases slightly (-0.16%) in the reform economy (this is due to the lower exit rate and weaker selection in the long run)²³, average weighted productivity in the economy increases since the

²³That is, by weaker selection we mean that less (more) low productivity firms exit (survive).

increase in the covariance between output shares and productivity (better allocative efficiency) more than compensates (a 1.95% rise) for the reduction in average (unweighted) productivity. After the reform, a larger fraction of production is placed with more productive firms. The change in credit terms between the benchmark and the reform economy allows firms to operate at a more efficient scale, so the correlation between capital stock and productivity increases.

7 Conclusion

We extend a standard model of firm dynamics to incorporate Chapter 7 and Chapter 11 bankruptcy choices. We find that if reforms proposed by legal and economic scholars are followed, there can be significant changes in borrowing costs, capital structure and investment decisions, as well as the cross-sectional distribution of firms. The general equilibrium consequences of such reforms can lead to a substantial rise in consumer welfare and moderate changes in measured TFP. The reforms are shown to yield benefits to allocative efficiency.

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Appendix

Appendix A1: Data

We use data from Compustat North America Fundamental Annual.²⁴ Our choice of firm identifier is GVKEY. The sample period for the fundamentals data ranges from 1980 to 2012. Our year variable is extracted from the variable DATADATE. We exclude financial firms with SIC codes between 6000 and 6999, utility firms with SIC codes between 4900 and 4999, and firms with SIC codes greater than 9000 (residual categories). Observations are deleted if they do not have a positive book value of assets or if gross capital stock or sales are either zero, negative, or missing. We censorize the top and bottom 2% of the ratios we construct as in Henessy and Whited [19]. The final sample is an unbalanced panel with more than 12,000 firms and 117,746 firm/year observations. All nominal variables are deflated using the CPI index (normalized to 100 in 1983). See Tables 7 and 8.

²⁴All variable names correspond to the Wharton Research Data Services (WRDS) version of Compustat.

Table 7: Variables

Variable	Item (old definition)	Description
GVKEY		Firm Identifier
DATADATE		Data Date
Company Name		
DLDTE		Research Company Deletion Date
DLRSN		Research Co Reason for Deletion
NAICS		
SIC		
AT	6	book assets
PPEGT	7	Property, Plant and Equipment - Total (Gross)
SPPE	107	Sale of Property
CAPXV	30	Capital Expend Property, Plant and Equipment
DP	14	Depreciation and Amortization
IB	18	Income Before Extraordinary Items
SSTK	108	Sale of Common and Preferred Stock (equity issuance)
DLTT	9	Long-Term Debt - Total
DLC	34	Debt in Current Liabilities
DVP	19	Dividends - Preferred/Preference
DVC	21	Dividends Common/Ordinary
PRSTKC	115	Purchase of Common and Preferred Stock
CHE	1	Cash and Short-Term Investments
SALE	12	Sales
CEQ	60	Common/Ordinary Equity - Total
PRCC_F	199	Price Close - Annual Fiscal
CSHO	25	Common Shares Outstanding
ACT	4	Current Assets - Total
LCT	5	Current Liabilities - Total
OIBDP	13	Operating Income Before Depreciation
XINT	15	Interest and Related Expense - Total
INVT	3	Inventories - Total
RECT	2	Receivables Total
BAST	104	Short Term Borrowings
PPENT	8	Property, Plant and Equipment - Total (Net)
DM	241	Debt - Mortgages and Other Secured = Secured Debt
DD1		Long Term Debt Due in one Year
LT		Total Liabilities
GP		Gross Profits
DT		Total Debt Including Current
TFVA		Total Fair Value Assets
TFVL		Total Fair Value Liabilities
EBIT		Earnings before Interest and Taxes
EBITDA		Earnings before Interest

Source: Compustat - Fundamental (WRDS).

Table 8: Derived Variables

Variable	Item (old definition)	Description
PPEGT + CHE	7 + 1	Total Assets = Capital + Cash
SSTK / (PPEGT + CHE)	108 / (7 + 1)	Equity Issuance / Total Assets
CAPXV-SPPE	30-107	gross investment
CAPXV-SPPE-DP	30-107-18	net investment
(CAPXV-SPPE-DP) / (PPEGT + CHE)	(30-107-18) / (7 + 1)	Net Investment / Total Assets
DVP+DVC+PRSTKC	19+21+115	Dividends = total cash distributions
	14+18	Cash Flow
(DVP+DVC+PRSTKC) / (PPEGT + CHE)	(19+21+115) / (7 + 1)	dividends / Assets
DLTT+DLC	34+9	Total debt
DLTT+DLC-CHE	34+9-1	Net Debt = Total Debt - Cash
DLTT+DLC-CHE _{i0}		Negative Net Debt
EBITDA/XINT		Interest Coverage Ratio (EBITDA)

Source: Compustat - Fundamental (WRDS).

Identifying Exit and Bankruptcies

We document firm exit, Chapter 11 bankruptcy and Chapter 7 bankruptcy using a set of different sources. We code a firm/year observation as being in Chapter 11 bankruptcy whenever the following happens:

- Footnote to total assets in period $t+1$ reports code “AG” (reflects adoption of fresh-start accounting upon emerging from Chapter 11 bankruptcy)
- Footnote to total assets in period t reports code “TL” (Company in bankruptcy or liquidation) and the bankruptcy event does not lead to firm deletion
- Footnote to total assets in period t reports code “TL” (Company in bankruptcy or liquidation), the bankruptcy event leads to firm deletion and the variable DLRSN (Research Company Reason for Deletion) is equal to codes 01 (Acquisition or merger), 02 (Bankruptcy), 04 (Reverse Acquisition), 07 (Other, no longer files with SEC among other possible reasons, but pricing continues), 09 (now a private company) and 10 (Other, no longer files with SEC among other possible reasons).
- If the firm/year observation corresponds to the last period of the firm in our sample, the variable DLRSN (Research Company Reason for Deletion) is equal to code 02 (Bankruptcy) and the footnote to assets does not contain bankruptcy information.

To complement the set of Chapter 11 bankruptcies we find using Compustat and dates from the footnote of Total Assets, we use Chapter 11 bankruptcy dates provided for firms with assets worth 100 million or more (in 1980 US\$) available in the UCLA-LoPucki Bankruptcy Research Database (BRD).

We code a firm/year observation as being in Chapter 7 bankruptcy whenever the following happens:

- Footnote to total assets in period t reports code “TL” (Company in bankruptcy or liquidation), the bankruptcy event leads to firm deletion and the variable DLRSN (Research Company Reason for Deletion) is equal to code 03 (Liquidation).
- If the firm/year observation corresponds to the last period of the firm in our sample, the variable DLRSN (Research Company Reason for Deletion) is equal to code 03 (Liquidation) and the footnote to assets does not contain bankruptcy information.

The classification into Chapter 11 and Chapter 7 bankruptcy during the last period of the firm in the sample is the same used by Duffie, Saita and Wang [15].

To be consistent with the definition of Chapter 11 bankruptcy, a deleted firm (i.e. a firm that disappears from our sample) is counted as a firm exit if the variable DLRSN is not equal to codes 01, 02, 04, 07, 09 or 10 (i.e. those not identified as continuing firms due to mergers, reorganization or because they go private for example). That implies that we classify a deletion as exit if the code equals code 02 (liquidation), codes 11 through 14 or if the code is missing. This is consistent with the definition of exit that the U.S. Bureau of the Census uses to construct its exit statistics. Table 9 provides summary statistics about the frequency of each of the above codes. Using this information, we have 173,617 nonbankrupt firm/year observations, 1,319 Chapter 11 firm/year observations, and 315 Chapter 7 firm/year observations. Table 9 presents a set of summary statistics. Moments in this table are computed as the time series average of the corresponding cross-sectional statistic.

Table 9: Bankruptcy, Deletion and Exit Statistics

Moment (%)	
Frequency of Deletion	7.29
Frequency of Deletion via Exit	1.1
Frequency of Deletion via M & A	3.48
Frequency of Deletion via going private	0.26
Frequency of Deletion via Chapter 7	0.184
Frequency of all Bankruptcy	0.91
Fraction of Chapter 7 Exit	58.52
Fraction of Chapter 11 Bankruptcy	78.94

Note: Moments are computed as time series averages of the cross-sectional statistic. Deletion corresponds to the fraction of firms that disappear from our sample in any given period. M& A refers to mergers and acquisitions. Source: Compustat - Fundamental (WRDS).

In Table 1 we include tests of the differences between means. To do so, for each variable

of interest x_t (i.e. variables listed in Table 1), we run the following regressions:

$$x_{it} = a_0 + a_1 d_{it}^{ch11} + a_2 d_{it}^{ch7} + b_t + u_{it} \quad (\text{A.1.1})$$

where d_{it}^{ch11} is a dummy variable that takes value 1 if the firm/year observation corresponds to the start of a Chapter 11 bankruptcy and zero otherwise; d_{it}^{ch7} is a dummy variable that takes value 1 if the firm/year observation corresponds to a Chapter 7 bankruptcy and zero otherwise; and b_t corresponds to a full set of year fixed effects. A significant coefficient a_1 reflects that average x_t is significantly different for firms in Chapter 11 bankruptcy than that of Non-bankrupt firms. Similarly, a significant coefficient a_2 reflects that average x_t is significantly different for firms in Chapter 7 bankruptcy than that of Non-bankrupt firms. To test whether that average of x_t is significantly different for firms in Chapter 7 than for those in Chapter 11 we run a similar regression using only observations in Chapter 7 and Chapter 11 and using as a regressor d_{it}^{ch7} and time fixed effects. A significant d_{it}^{ch7} coefficient reflects means between these two groups are significantly different.

Z-scores and Distance to Default

The Altman z-score is a commonly used measure of the level of distress of corporations (see Altman [1] for the seminal paper on the subject). The basic idea is to construct an index based on observable variables that helps to predict whether a firm is close to bankruptcy or not. More specifically, the z -score is defined as follows:

$$z = 1.2x_1 + 1.4x_2 + 3.3x_3 + 0.6x_4 + 0.999x_5,$$

where x_1 is the working capital to total assets ratio (measured as current assets minus current liabilities over assets), x_2 is retained earnings over assets, x_3 corresponds to the earnings before interest and taxes over assets, x_4 is the market value of equity over the book value of total liabilities and x_5 is sales over total assets. The coefficients are determined using a multiple discriminant statistical method. Once the z -score is constructed the rule of thumb is to define all firms having a z -score greater than 2.99 as “non-distressed” firms, while those firms having a z -score below 1.81 as “distressed” firms. The area between 1.81 and 2.99 is defined as the “zone of ignorance”.

In order to construct a default probability based on the distance to default model, we follow Duffie et. al [15]. The default probability is constructed using the number of standard deviations of asset growth by which a firm’s market value of assets exceeds a liability measure. That is, for a given firm, the 1-year horizon distance to default is defined as

$$D_t = \frac{\ln(V_t/L_t) + (\mu_A - 1/2\sigma_A^2)}{\sigma_A}$$

where V_t is the market value of the firm's assets at time t and L_t is the liability measure (calculated as short term book debt plus 1/2 of long-term book debt), μ_A is the mean rate of asset growth and σ_A the standard deviation of asset growth. The market value of the firm is estimated following the theory of Merton (1974) and Black and Scholes (1973). More specifically, we let W_t denote the market value of equity which is equal to an option on the value of a firm's assets, currently valued at V_t , with strike price of L_t and one-year to expiration. We obtain the asset value V_t and the volatility of asset growth by solving the following system of equations iteratively:

$$\begin{aligned} W_t &= V_t \Phi(d_1) - L_t e^r \Phi(d_2), \\ \sigma_a &= \text{Std.Dev.}(\ln(V_t) - \ln(V_{t-1})), \\ d_1 &= \frac{\ln(V_t/L_t) + (r + 1/2\sigma_A^2)}{\sigma_A}, \\ d_2 &= d_1 - \sigma_A, \end{aligned}$$

where $\Phi(\cdot)$ is the standard normal cdf, Std. Dev. denotes standard deviation and r is the risk-free rate (that we take to be the real 1-year T-bill rate). The initial guess for V_t is the sum of W_t (measured as end-of-period real stock price times number of shares outstanding) and the book value of total debt (sum of short term and long term book debt). Once V_t and σ_A are estimated, we compute D_t . The corresponding default probability is

$$p_t^D = \Phi(-D_t).$$

Appendix A2: Computational Algorithm

In this section, we describe our computational algorithm.

1. Set grids for $k \in K$, $b \in B$ and $z \in Z$.
2. Guess initial wage rate w^0 , price schedule $q^0(k', b', z)$, recovery rate schedule $\phi^0(k', b', z)$.
3. **Solve Firm Problem:** Given the bond price schedule, recovery schedule and wage rate, solve the firm problem to obtain capital, debt, exit and bankruptcy decision rules as well as value functions.
4. **Update Recovery Schedule:** Using the value functions obtained in point 3, solve the renegotiation problem to obtain $\phi^1(k', b', z)$.
5. **Update Bond Price Schedule:** Using the exit and bankruptcy decision rules obtain a price function that is consistent with them. Let it be $q^1(k', b', z)$.
6. If $\|\phi^1(k, b, z) - \phi^0(k, b, z)\| < \epsilon_\phi$ and $\|q^1(k', b', z) - q^0(k', b', z)\| < \epsilon_q$, for small ϵ_ϕ and ϵ_q , then we have obtained the equilibrium price and recovery schedule (for a given price

w^0), continue to the next step. If not, update the price and recovery schedule (i.e. set $\phi^0 = \phi^1$ and $q^0 = q^1$) and return to step 3.

7. **Update wage using Free Entry Condition:** Evaluate the free entry condition V^E at w^0 . If it holds with equality, continue. If it does not, proceed as follows:

- If V^E is positive, increase w^0 and return to point 3.
- If V^E is negative, reduce w^0 and return to point 3.

8. **Derive Equilibrium Mass of Firms from Labor Market Clearing:**

- Set $M = 1$ and compute the stationary distribution associated with the set of decision rules obtained above and this mass of entrants. Denote this distribution $\hat{\Gamma}(k, b, z; M = 1)$.
- Calculate labor demand $\hat{\Gamma}(k, b, z; M = 1)$, that is

$$\hat{N}(M = 1) = \int n(z, k, b) d\hat{\Gamma}(z, k, b; M = 1).$$

- Set M^0 to satisfy the labor market clearing condition. That is set M^0 as follows

$$M^0 = 1/\hat{N}(M = 1)$$

- The equilibrium prices and distribution are: $w^* = w^0$, $M^* = M^0$, $\Gamma^* = M^* \hat{\Gamma}(k, b, z; M = 1)$, $q^* = q^0$, $\phi^* = \phi^0$.
- **Aggregates and Taxes:** Compute aggregate consumption and taxes.